Hedge Fund Performance and Higher-Moment Market Models

Angelo Ranaldo [α,ρ], Swiss National Bank[*]

Laurent Favre [β], AlternativeSoft.com[*]

Abstract

The CAPM model is hard put to explain the superior performance of hedge funds in the past. We argue that the Markowitz mean-variance criterion underpinning the traditional CAPM may fail to capture systematic features characterizing hedge fund performance. Thus, we extend the two-moment market model to a higher-moment model to accommodate coskewness and cokurtosis. The higher-moment approach is more appropriate for capturing the non-linear relation between hedge fund and market returns and accounting for the specific risk-return payoffs of each hedge fund investment strategy. The key result is that the use solely of the two-moment pricing model may be misleading and may wrongly indicate insufficient compensation for the investment risk.

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[α] Angelo Ranaldo, Swiss National Bank, Research Department, Börsenstrasse 15, P.O. Box 2800, Zurich, Switzerland. E-mail: angelo.ranaldo@snb.ch, Phone: ++41.44.6313826, Fax: ++41.44.6313901.

[ρ] Contact author: Angelo Ranaldo.

[β] Laurent Favre, AlternativeSoft AG, www.alternativesoft.com, Tannenrauchstrasse 55, 8032 Zurich, Switzerland.

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"Betters love skewness, not risks, at the horse track!"
Golec and Tamarkin (1998)

1. Introduction

The hedge fund industry has grown impressively in the last decade. Assets under management[1] rose from $170 billions in 1995 to more than 1 trillion in January 2005. Many studies show that hedge funds have a superior performance and that the introduction of hedge funds in a classical portfolio enhances its performance. This superior performance is generally ascribed to the skills of the hedge fund managers. Some scepticism remains, however. First, hedge fund indices are broadly affected by the survivorship and performance measurement bias. Second, the nature of the return-generating process in hedge funds remains an unresolved issue. The attractive performance of hedge funds may be due to inadequate techniques for measuring the risk-return profile of hedge funds. The main aim of this research is indeed to investigate how to price hedge fund returns and, in particular, the validity of the traditional asset pricing models in measuring the risk-return trade-off in hedge fund investments.

The Sharpe-Lintner-Mossin equilibrium model, usually called the Capital Asset Pricing Model (hereafter called CAPM), is the most commonly used asset pricing model. This particular theoretical framework restricts the risk-return trade-off to a simple mean-variance relationship and / or to a quadratic utility function. However, the empirical evidence shows that the normality hypothesis has to be rejected for many hedge fund returns. Furthermore, a quadratic utility function for an investor implies an increasing risk aversion[2]. Instead, it is more reasonable to assume that risk aversion decreases with an increase in wealth. In this paper, we consider some extensions of the traditional mean-variance framework that account for higher moment conditions and a more variegated structure of the risk premium concept. In particular, we examine the role of coskewness and cokurtosis in pricing hedge fund investments.

Skewness characterizes the degree of asymmetry of a distribution around its mean. Positive (negative) skewness indicates a distribution with an asymmetric tail extending towards more positive (negative) values. Kurtosis characterizes the relative peakness or flatness of a distribution compared with the normal distribution. Kurtosis higher (lower) than three indicates a distribution more peaked (flatter) than a normal distribution.. Similarly to the so-called systematic risk or beta, it is possible to test for a systematic skewness and systematic kurtosis. Systematic skewness and kurtosis are also called coskewness and cokurtosis (Christie-David and Chaudry [2001]). Provided that the market has a positive skewness of returns, investors will prefer an asset with positive coskewness. Cokurtosis measures the likelihood that extreme returns jointly occur in a given asset and in the market. Investors prefer small cokurtosis.

The common characteristic of the models accounting for coskewness and cokurtosis is that they incorporate higher moments in the asset pricing framework. In the literature, two main approaches have been investigated: three-moment and four-moment CAPM. The theoretical basis for three-moment CAPM was developed in Kraus and Litzenberger [1976], Ingersoll [1975], and Jurcenzko and Maillet [2002]. Other authors empirically study the three-moment CAPM. Barone-Adesi [1985] proposes a quadratic model to test the three-moment CAPM. Harvey and Siddique [2000] find that the systematic skewness requires an average annual risk premium of 3.6% for US stocks. They also find that portfolios with high systematic skewness are composed of winner stocks (momentum effect). Harvey [2000] shows that skewness, coskewness and kurtosis are priced in the individual emerging markets but not in developed markets. He observes that volatility and returns in emerging markets are significantly positively related. But the significance of the volatility coefficient disappears when coskewness, skewness, and kurtosis are considered. Harvey's explanation for this phenomenon is the low degree of integration of the emerging markets[3].

Berenyi [2002], Christie-David and Chaudry [2001], Chung, Johnson and Schill [2001], Hwang and Satchell [1999], Jurczenko and Maillet [2002], Galagedera, Henry and Silvapulle [2002] propose the use of the Cubic Model as a test for coskewness and cokurtosis. Berenyi [2002] applies the four-moment CAPM to mutual fund and hedge fund data. He shows that volatility is an insufficient measure of risk for hedge funds and for medium risk averse agents. Christie-David and Chaudry [2001] employ the four-moment CAPM on the future markets. They show that systematic skewness and systematic kurtosis increase the explanatory power of the return generating process of future markets. Hwang and Satchell [1999] investigate coskewness and cokurtosis in emerging markets. They show that systematic kurtosis is better than systematic skewness in explaining emerging market returns. Chung, Johnson and Schill [2001] compare the four-moment CAPM with the Fama-French two factors model.

The main question we raise in this paper is whether the first two moments are enough to fully explain the risk-return characteristics of hedge funds. This question can be paraphrased as follows: Are hedge fund index returns linearly or non-linearly related to market returns. Then, we extend the two-moment market model in the three-moment and four-moment implementation. These less restrictive forms of the traditional mean-variance model accommodate systematic volatility (i.e. beta), systematic skewness, and systematic kurtosis. Finally, we compute the required rate of return for hedge fund indices in the spirit of linear and non–linear pricing models.

Our paper is organized as follows. In Chapter 2, we explain the economic arguments behind coskewness and cokurtosis. In Chapter 3, we derive the three-moment and four-moment CAPM models from the expected utility function. In Chapter 4, we empirically analyse whether beta, coskewness, and cokurtosis affected istorical returns on the hedge fund index. Finally, we set out our concluding remarks.

2. Arguments for the existence of coskewness and cokurtosis

The existence of skewness and kurtosis in asset return distributions is well known. Here, the research focus is rather on the existence of coskewness and cokurtosis and, if any, their relevance in modelling asset pricing. The source of coskewness and cokurtosis in asset return distributions is essentially twofold. On the one hand, peculiar return distribution patterns may originate from the use of specific trading strategies. Hedge fund managers pursue varied hedging and arbitrage strategies generating pay-off profiles that are extremely different from those of traditional assets. Moreover, hedge fund trading is also characterised by the use of leverage and derivatives. Agarwal and Naik [2000, 2002], for instance, show that writing and buying at-the-money and out-of-the money options increases significantly the explanatory power of hedge fund returns. On the other hand, skewed and / or kurtotic return distributions may be seen as the statistical expression of market inefficiency and market frictions. Specifically, non-normal return distributions may be due to illiquidity and low information transparency. Hedge fund managers frequently base their strategies on highly illiquid and / or volatile assets. Illiquidity also means a high barrier to entry that prevents many retail investors from trading in hedge funds[4]. Hedge fund investments are also characterised by opaque and asymmetric information. Typically, hedge fund trading strategies imply disguising trading positions since a full and transparent information disclosure would jeopardise their trading opportunities. Factors such illiquidity and information opaqueness contrast with the assumptions underpinning the standard CAPM model[5].

3. The four-moment CAPM

In this section, we develop the four-moment CAPM. It is worth emphasising that we will limit our empirical investigation to a time series analysis. We do not apply the two-pass method (time series and cross-sectional analysis) as in Fama-MacBeth (1973) to empirically estimate the validity of the CAPM. Thus, our analysis does not represent an inference for equilibrium or expected returns.

The four-moment CAPM represents a pricing model for the beta, coskewness and cokurtosis of the hedge fund strategies. We use i to denote a generic asset and m the reference market. R_i and R_m denote their respective returns. The investment problem for an investor is to maximise the expected utility at the end of the period. The investor's expected utility can be represented as a Taylor series expansion[6] of order n:

$$\begin{aligned} E[U(R_i)] &= U[E(R_i)] + \frac{1}{2!}U''[E(R_i)]E[R_i - E(R_i)]^2 + \frac{1}{3!}U'''[E(R_i)]E[R_i - E(R_i)]^3 \\ &+ \frac{1}{4!}U''''[E(R_i)]E[R_i - E(R_i)]^4 + \sum_{n=5}^{\infty}\frac{1}{n!}U^n[E(R_i)]E[R_i - E(R_i)]^n \end{aligned} \qquad (1)$$

In a compacted form and limiting the series to the fourth order, we obtain:

$$E[U(R_i)] \approx U[E(R_i)] + \frac{1}{2!}U''[E(R_i)]\sigma^2(R_i) + \frac{1}{3!}U'''[E(R_i)]S^3(R_i) + \frac{1}{4!}U''''[E(R_i)]K^4(R_i) \quad (2)$$

with

$$\begin{aligned} \sigma &= \left(\mathrm{E}\left(\mathrm{R_i} - \overline{\mathrm{R}}_\mathrm{i}\right)^2\right)^{1/2} \\ \mathrm{S} &= \left(\mathrm{E}\left(\mathrm{R_i} - \overline{\mathrm{R}}_\mathrm{i}\right)^3\right)^{1/3} \\ \mathrm{K} &= \left(\mathrm{E}\left(\mathrm{R_i} - \overline{\mathrm{R}}_\mathrm{i}\right)^4\right)^{1/4} \end{aligned} \qquad (3)$$

where $\overline{\mathrm{R}}_\mathrm{i}$ is the expected return of the asset i, σ is the volatility, S is the third moment, K is the fourth moment and U^n is the n^th derivative of the utility function U. In this paper, the terms S and K stand for third and four centred moments of the return distribution and not for skewness and kurtosis. In statistics, skewness and kurtosis are the third and fourth moments standardised respectively by the cube of the volatility and volatility to the power of four.

Assuming an indirect utility function (let us call it V), Jurcenzko and Maillet [2002] show that the financial market equilibrium for the four-moment CAPM between security i and the market portfolio m can be set by maximising equation (1) that is given by[7]

$$E(R_i) - R_f = \alpha_1\beta_{i,m} + \alpha_2\gamma_{i,m} + \alpha_3\delta_{i,m} \qquad (4)$$

With $\alpha_1 = \theta_2\sigma^2(R_m)$, $\alpha_2 = \theta_3 S^3(R_m)$, $\alpha_3 = \theta_4 K^4(R_m)$, $\beta = \Omega\omega / \sigma^2(R_m)$, $\gamma = \Sigma_m / S^3(R_m)$, and $\delta = \Gamma_m / K^4(R_m)$. $\theta_2 = -2V^{II}/V^I$, $\theta_3 = -3V^{III}/V^I$, and $\theta_3 = -4V^{IV}/V^I$, where V^N is the N-derivative of function. $\mathrm{R_f}$ signifies the return on a risk-free asset. α_1, α_2 and α_3 are risk premiums. $\beta_{i,m}$, $\gamma_{i,m}$ and $\delta_{i,m}$ are respectively the systematic beta, the systematic skewness and the systematic kurtosis V. Ω, Σ, and Γ are respectively the variance-covariance matrix, the vector of coskewness and the vector of cokurtosis between securities and market portfolio returns. As in the two-moment CAPM, where systematic beta is priced, the assumption in this four-moment CAPM is that the systematic skewness and systematic kurtosis are also priced. We expect a positive risk premium for positive beta since investors require a higher return for a higher beta. We expect a negative risk premium for positive systematic skewness since, in equilibrium, investors require a lower return for less downside risk. We expect a positive risk premium for positive systematic kurtosis since investors requires a higher return for assets with higher probability of extreme price co-variations with the market.

In equation (4), the three alphas represent respectively the market price, or risk premium, for an increase in beta, a decrease in systematic skewness, and an increase in systematic kurtosis. These three alphas are given by[8]

$$\alpha_1 = \frac{dE(R_i)}{d\sigma^2(R_i)}\sigma^2(R_m) \qquad \alpha_2 = \frac{dE(R_i)}{dS^3(R_i)}S^3(R_m) \qquad \alpha_3 = \frac{dE(R_i)}{dK^4(R_i)}K^4(R_m) \tag{5}$$

Adding equations (5) in equation (4), the four-moment CAPM is as follows:

$$E(R_i) - R_f = \frac{dE(R_i)}{d\sigma^2(R_i)}\sigma^2(R_m)\beta_{i,m} + \frac{dE(R_i)}{dS^3(R_i)}S^3(R_m)S_{i,m} + \frac{dE(R_i)}{dK^4(R_i)}K^4(R_m)K_{i,m} \tag{6}$$

The four-moment CAPM in equation (4) is the combination of the systematic beta, systematic skewness, and systematic kurtosis with the respective market prices alphas. If the investor prices $\beta_{i,m}$, $\gamma_{i,m}$ and $\delta_{i,m}$, the alpha values, α_1, α_2, α_3, should be significantly different from zero. Kraus and Litzenberger [1976] show how to calculate $\beta_{i,m}$ and $\gamma_{i,m}$ from the estimated coefficients α_1 and α_2 coming from the quadratic model. Here, we extend this method to the cubic model to obtain $\beta_{i,m}$, $\gamma_{i,m}$ and $\delta_{i,m}$ for the four-moment CAPM model. Using the procedure described in Appendix 1, we find that[9]:

$$\begin{aligned}
\hat{\beta}_{i,m} &= \hat{\alpha}_1 + \hat{\alpha}_2 \frac{S^3(R_m)}{\sigma^2(R_m)} + \hat{\alpha}_3 \frac{K^4(R_m)}{\sigma^2(R_m)} \\
\hat{\gamma}_{i,m} &= \hat{\alpha}_1 + \hat{\alpha}_2 \frac{K^4(R_m) - \left[\sigma^2(R_m)\right]^2}{S^3(R_m)} + \hat{\alpha}_3 \frac{m^5(R_m) - \sigma^2(R_m)S^3(R_m)}{S^3(R_m)} \\
\hat{\delta}_{i,m} &= \hat{\alpha}_1 + \hat{\alpha}_2 \frac{m^5(R_m) - \sigma^2(R_m)S^3(R_m)}{K^4(R_m)} + \hat{\alpha}_3 \frac{m^6(R_m) - \left[S^3(R_m)\right]^2}{K^4(R_m)}
\end{aligned} \tag{7}$$

where m^5 and m^6 stand for the fifth and sixth centred moment of the return distribution. Thus, the estimates of α_1, α_2, and α_3 can be translated to beta $\beta_{i,m}$, systematic skewness $\gamma_{i,m}$ (or coskewness), and systematic kurtosis $\delta_{i,m}$ (or cokurtosis).

4. Empirical tests

4.1. Statistics for hedge fund indices

The data used are the monthly returns on 16 hedge fund indices obtained from the HFR database from January 1990 to August 2002. In these 16 hedge fund indices, the index components are equally weighted. We use a market portfolio composed of 70% of the Russell 3000 index and 30% of the Lehman US aggregate bond index. This market portfolio is in line with the previous literature[10] and with the idea that hedge fund managers typically have the highest trading exposure on equity and bond markets. Consistent with this reasoning, two different market portfolios are used for the Relative Value and Fixed Income Arbitrage hedge fund indices. In the former portfolio, the Wilshire all growth index replaces the Russell 3000 index. In the latter, we use the Merrill Lynch high yield US corporate index. All the indices are in USD. The risk free rate is the US 1 month Certificate of Deposit.

Table I shows the descriptive statistics for returns on hedge fund indices and a market portfolio.

[Insert Table I here]

Over the 12 years considered in our sample period, the top-performing index has been the equity hedge with an annualised return of 17%. However, all the hedge fund indices have very attractive performances. In fact, except for the Short Seller index, all the hedge funds outperform the market portfolio. The wide range of standard deviations implies major differences between the hedge fund trading strategies. In particular, the risk related to Short Sellers, Equity Non-Hedged and Emerging Markets is higher than the portfolio risk (10.7% on an annualised basis). Based on the Jarque-Berra test, we cannot accept the normal hypothesis at the 95% significance level for the monthly return distributions of all hedge fund indices[11].

Skewness in the return distribution is negative for 11 out of the 16 hedge fund indices. This suggests that extreme negative price falls are more likely than extreme price increases for most of the hedge fund indices analysed. We also observe a high probability of extreme price variations in Distressed Securities, Event Driven, Fixed Income Arbitrage, Merger Arbitrage, and Relative Value where the excess kurtosis is rather high. The index with the highest probability of loss is Short Seller. This shows a 1% probability of losing more than -16.2% every month when volatility, skewness and kurtosis are taken into account[12]. The index with the largest monthly loss over the sample period is indeed the Short Seller with a price drop of -21.2%.

4.2. The market model

One of the more important developments in modern capital market theory is the Sharpe-Lintner-Mossin mean-variance equilibrium model, commonly called the Capital Asset Pricing Model (CAPM)[13]. This model predicts that the expected excess return from holding an asset is proportional to the covariance of its return to the market portfolio, i.e. the beta. As underlined by Merton [1990], despite the significant impact which this model had on the academic and non-academic financial community, it is still subject to theoretical and empirical criticism. The model assumes that investors choose their portfolios according to the Markowitz mean-variance criterion. The list of the assumptions and conditions necessary for the validity of the mean-variance analysis is rather demanding[14]. The mean-variance criterion is not consistent with the von Neumann-Morgenstern axioms of choice unless asset prices have Gaussian probability distributions or investor preferences are quadratic.

The main objective of this study is to empirically analyse the return generating function for hedge funds and to verify whether hedge fund index returns are related linearly or non-linearly to market returns. Although this analysis does not represent an empirical test for the CAPM, it does involve examining the two-moment criterion. To do this, we perform ex-post empirical tests using observed values. The ex-post approach is commonly used as a yardstick for measuring past portfolio performance. On the basis of the third and fourth order polynomial utility function described above, we will extend the Market Model to the Quadratic and Cubic Model. Let us start with the Market Model, which can be expressed as follows:

$$R_{i,t} - R_{f,t} = \alpha_0 + \alpha_1 \left(R_{m,t} - R_{f,t} \right) + \varepsilon_t \qquad (8)$$

The Market Model is used as the benchmark model to compare the more general asset pricing frameworks represented by the Quadratic and Cubic Models. The Market Model is a linear equation that relates the equilibrium expected return on each asset to a single identifiable risk measure. In other words, the hedge fund return is linked to the market risk premium with its beta.

Throughout this research, we employ the Least Square method to estimate the regression parameters of the multi moment market models[15]. We account for heteroskedasticity and residual autocorrelation by using Newey-West adjustments for standard error and covariance. The parameters are estimated using the whole sample period. It is arguable whether these parameters are constant over time. We test the coefficients' stability for the three pricing models, i.e. the Market, Quadratic and Cubic models. Recursive and rolling-sample regressions show that estimates steadily converge to their final value[16]. However, few extreme events have an observable influence. For instance, the financial markets crisis around August 1998 has an impact on the estimated parameters. Such price disruptions are inherently part of the financial markets' dynamics. Therefore, we take these extreme price movements into account in our estimation period.

Table II shows the regression results of the Market Model. This table shows at least two interesting results. First, the regression intercept (i.e. coefficient α_0 in regression (8)) is always positive and significant, except for the Short Seller index. This pervasive evidence could mean that hedge funds systematically outperform the market. But this interpretation may be wrong if the hedge fund and market returns are non-linearly related. Second, the size of the regression slopes (i.e. coefficient α_1) is very diverse across the sample. The beta coefficients show that covariances of hedge fund returns with the market portfolio differ greatly. In particular, Emerging Markets and Equity Non-Hedge indices have beta coefficients of around one whereas other hedge funds (e.g. Convertible Arbitrage and Equity Market Neutral) have small but significant betas and the Short Seller index has a negative beta. It is noteworthy that the CAPM model predicts that the highest beta assets are supposed to provide the highest expected excess returns. This is not the case for hedge funds. In fact, the Macro hedge fund index has a relatively low beta (i.e. $\alpha_1=0.37$) but the highest performance (i.e. 17% on annual basis). These considerations call for pricing models that go beyond the traditional two-moment CAPM model.

[Insert Table II here]

4.3. Quadratic Model

The Quadratic Model extends the pricing relation to the third moment. This approach assumes that economic agents take into consideration the skewness of return distributions. There is a departure from normality and evidence of skewed return distribution in several asset markets, including derivatives markets, structured portfolios, and indeed hedge funds. The Quadratic Model states that the relation between an asset and the market portfolio is quadratic. It can be expressed as follows:

$$R_{i,t} - R_{f,t} = \alpha_0 + \alpha_1 \left(R_{m,t} - R_{f,t} \right) + \alpha_2 \left(R_{m,t} - E\left(R_m \right) \right)^2 + \varepsilon_t \qquad (9)$$

There is an intuitive link between higher-moment pricing models and multi-factor models. In the most common setting of the Arbitrage Price Theory (APT), for instance, there is more than one factor that systematically determines the asset value. It is generally assumed that the unexpected component of these factors determines the asset value. We can apply this reasoning to the Quadratic Model in equation (9). Besides α_1 , which accounts for the systematic risk, α_2 captures the asset value's sensitivity to the squared deviation of the market portfolio's return from its expected value. Table III reports regression results from the Quadratic Model.

[Insert Table III here]

The coefficient of determination R^2 is defined as the percentage of the total variation in the dependent variable explained by the independent variables. Table III shows the comparison of the adjusted R^2 from the Market Model and Quadratic Model. We observe that using the Quadratic Model the adjusted R^2 substantially increases for more than half of the hedge funds analysed. This result is supported by the Akaike Information Criterion statistics. In particular, we observe a substantially higher adjusted R^2 for Convertible Arbitrage, Distressed Securities, Event Driven, Emerging Markets, Fund of Funds, Market Timing, Merger Arbitrage, Relative Value Arbitrage and Weighted Composite Index. Most of these strategies are exposed to event risks.

The increase in the coefficient of determination is combined with a high significance level of the coefficient α_2 in equation (9), which is a proxy for the existence of coskewness. All these hedge fund indices have a non-linear relation with the market portfolio. This non-linear relation shows that these hedge funds will increase or decrease market portfolio skewness if added to the market portfolio. Indeed, the hedge fund strategies with negative α_2 coefficients have concave payoffs with respect to their market portfolio. This is the case for most of the hedge funds. Conversely, hedge funds with a positive coskewness coefficient have a convex payoff (as in the case of Fixed Income Arbitrage, Merger Arbitrage and Statistical Arbitrage). Negative (positive) coskewness means that hedge funds tend to have an asymmetric tail extending towards more negative (positive) returns with respect to the distribution of market portfolio returns.

Transforming α_1 and α_2 as described in equation (7), we obtain beta (systematic risk due to the covariance between hedge fund and market portfolio returns) and gamma (systematic risk due to the coskewness between hedge fund and market portfolio returns) implied by the Quadratic Model estimation. Table III reports these values. Comparing beta values in Table II and III, we observe that all the betas implied by the Quadratic Model are smaller than those from the Market Model. The only exception is the Merger arbitrage strategy. This result suggests that part of the explanatory power attributed to beta in the Market Model has been taken over by α_2 and, consequently, by coskewness.

4.4. Cubic Model

The Cubic Model extends the Market Model by including squared and cubic market returns as additional factors. This extension allows us to test the role of coskewness and cokurtosis in the asset pricing process. As underlined by Barone-Adesi, Gagliardini, and Urga [2002], the Cubic Model does not permit a precise estimation of the coskewness and cokurtosis risk premia. However, it provides a powerful test of the relationship between risk and expected return implied by the asset pricing model in equation (2). The Cubic Model is described by

$$R_{i,t} - R_{f,t} = \alpha_0 + \alpha_1\left(R_{m,t} - R_{f,t}\right) + \alpha_2\left(R_{m,t} - E\left(R_m\right)\right)^2 + \alpha_3\left(R_{m,t} - E\left(R_m\right)\right)^3 + \varepsilon_t \qquad (10)$$

Equation (10) assumes that asset returns are a function of a polynomial expansion of the market return up to the fourth order. In this Cubic Model, α_0 is the asset intercept, α_1 represents the asset price's sensitivity to excess returns of the market portfolio, α_2 and α_3 its sensitivity to the market portfolio's squared and cubed returns, respectively. In this line of reasoning, α_2 and α_3 convey coskewness and cokurtosis in the hedge funds' return-generating process.

[Insert Table IV here]

Table IV shows the regression results of the Cubic Model. In general, we observe that coefficient α_3 has a weak explanatory significance. This evidence suggests that systematic kurtosis plays a minor role in pricing the risk profile of hedge funds. In fact, the cokurtosis coefficient is only significant for four strategies: Convertible Arbitrage, Emerging Market, Market Timing, and Merger Arbitrage. A positive cokurtosis coefficient, like in the Convertible Arbitrage, Market Timing, and Emerging Market case, means that the hedge fund index is adding kurtosis to the market portfolio. Hence, the insertion of these hedge fund indices into the market portfolio will strengthen the likelihood of extreme returns. In contrast, Merger arbitrage has a negative cokurtosis coefficient, meaning that the addition of this specific hedge fund index in the market portfolio will decrease the market portfolio kurtosis[17].

Whether the four-moment market model is the appropriate asset pricing model for Convertible Arbitrage, Emerging Markets, and Merger Arbitrage is an open question. According to Hwang and Satchell [1999], the co-moments in the four-moment CAPM may have collinearities limiting the power of testing the model. It is possible that for these two hedge funds there is a spurious collinearity between the systematic skewness and systematic kurtosis. Comparing the Quadratic Model (Table III) and the Cubic Model (Table IV), we observe that the high significance level of the cokurtosis coefficient comes at the cost of a decrease in the significance level of the coskewness coefficient. The t-statistics of the coskewness coefficients for Convertible Arbitrage, Emerging Markets, Market Timing and Merger Arbitrage significantly decrease from Table III to Table IV.

Table IV also compares the adjusted R^2 coming from the estimation of the Market Model, Quadratic Model, and Cubic Model. We note that the Cubic Model provides a slightly higher coefficient of determination for Convertible Arbitrage, Emerging Market, Market Timing, and Merger Arbitrage.

4.5. Required rate of return of hedge fund indices

The last issue we address in this research is the estimation of the required rate of return for hedge funds. The required rate of return is defined as the investor's compensation for risk. The extent of that risk remuneration depends on the relationship between the equilibrium expected return on each hedge fund and the identifiable risk measure. As emphasised above, our analysis does not provide an estimate for equilibrium returns. Here, we assume that expected returns, beta, systematic skewness and kurtosis are simply determined on the basis of historical data. The main idea is to show that different asset pricing models identify three different risk definitions. Thus, the required rate of return stemming from the Market, Quadratic, and Cubic Model may lead to different required rates of return. In particular, we expect the following:

- A rational investor dislikes (prefers) negative (positive) coskewness. Thus, comparing the Market and Quadratic Model, we expect that the expected rate of return increases (decreases) for those hedge funds with a significantly negative (positive) coskewness coefficient, i.e. α_2 from equation (9).
- A rational investor dislikes (prefers) positive (negative) cokurtosis. Thus, comparing the Quadratic and Cubic Model, we expect that the expected rate of return increases (decreases) for those hedge funds with a significantly positive (negative) cokurtosis coefficients, i.e. α_3 from equation (10).

The procedure for deciding on the appropriate multi-moment market model is as follows. The Akaike Information Criterion (AIC) is computed for the Market Model, the Quadratic Model and the Cubic Model. AIC is similar to the R^2 in that it rewards good fit but penalises the loss of degrees of freedom. Whichever of the Market, Quadratic and Cubic models that has the lowest AIC is selected.

The calculation of the required rate of return is as follows. We use the estimated coefficients from Tables II, III, and IV to calculate β (beta), γ (coskewness), and δ (cokurtosis), as described in equation (7) and appendix 1. Two scenarios are analysed. The former is based on the historical market performances and the latter is a hypothetical and more conservative scenario. In both cases, the expected market portfolio (i.e. $\overline{R}_m$) and free-risk returns are represented by the historical average values from Table I, respectively 7.6% and 4.6% per annum. In the first scenario, we assume that, in equilibrium, realized returns equal expected returns (i.e. $\overline{R}_m = R_{m,t}$ =7.6%). Note that even if realised and expected returns are equal, required rates of return differ along with the pricing model. In fact, each pricing model provides different estimates of α_1, α_2, α_3 which, in turn, imply dissimilar values for β, γ, and δ. These differences entail different required rates of return even when realised returns and expected returns are identical. For instance, comparing Table II and III, we see that the β values implied by the Market Model differ from those derived from the Quadratic Model. This means that the required rates of return stemming from the two pricing models will be different. In the second scenario, we assume instead that the realised market risk premium differs from the historical one (market portfolio return: 5.6%; free-risk asset return: 4.6%). This assumption implies a negative unexpected market portfolio return of (5.6%-7.6%)=-2% on a yearly basis.

As a concrete example, let us look at the calculation of the required rate of return for the Event Driven hedge fund strategy in Table V. The AIC criterion indicates that the appropriate model for Event Driven index is the Quadratic implementation (see the third row in Table V). The required rate of return according to the first scenario (i.e. realised and expected market portfolio returns of 7.6% and risk free asset return of 4.6%) results from the following calculation:

$$RRR_{monthly} = 0.010 + 0.442 * \left(7.6\% / 12 - 4.6\% / 12\right) + 0.953 * (7.6\% / 12 - 7.6\% / 12)^2 = 1.146\%$$
$$RRR_{annualized} = 1.146\% * 12 = 13.748\%$$

where 0.442 and 0.953 are beta and gamma according to the Quadratic Model (see Table III). The second parenthesis is zero because of the equivalence between realised and expected market portfolio returns. This equivalence apparently cancels out the systematic coskewness premium. However, the two-, three- and four-moment market models deliver different estimated coefficients and, therefore, diverse systematic risk components and required rates of return (in the case of the Event Driven strategy, for instance, the beta implied by the Market Model is 0.445 instead of 0.442 on the basis of the Quadratic Models; see Table II and III, respectively).

In the second scenario, the realised market portfolio return is 5.6%. Thus, the required rate of return for Event Driven hedge funds is equal to:

$$RRR_{monthly} = 0.010 + 0.442 * \left(5.6\% / 12 - 4.6\% / 12\right) + 0.953 * (5.6\% / 12 - 7.6\% / 12)^2 = 1.074\%$$
$$RRR_{annualized} = 1.074\% * 12 = 12.887\%$$

Here, the second parenthesis is different from zero and, therefore, the coskewness premium applies.

Table V shows the empirical findings on the required rate of return for all hedge fund strategies. Three main results emerge. First, the Market Model appears to be the most appropriate model for Equity Hedge, Equity Market Neutral, Fixed Income Arbitrage, Macro, Statistical Arbitrage and Short Seller index. For these hedge funds, the beta seems a comprehensive measure of risk. Hence, investing in these hedge funds does not require a risk premium for coskewness and cokurtosis.

Second, the results on coskewness support the hypothesis that rational investors dislike negative coskewness and prefer positive coskewness. In fact, in Table V we observe a higher required rate of return exactly for those hedge funds with negative estimates of α_2 both in the Quadratic and Cubic Model (see Table III and IV). In particular, this is the case for Convertible Arbitrage, Distressed, Event-Driven, Emerging Markets, Equity Non-Hedge, Fund of Funds, Market Timing, Relative Value and Weighted Composite. For these hedge funds, coskewness has a relevant contribution in the investment return-risk profile and commands a higher risk premia. The highest additional premium due to coskewness is for the Distressed (5.21%) and Event Driven (4.94%) indices. In fact, investing in these two hedge funds would require at least a 4% return for negative coskewness.

[Insert Table V here]

Third, in Table V, the results for the Convertible Arbitrage, Emerging Market, Fund of Funds, Market Timing and Merger Arbitrage indices support the hypothesis that rational investors dislike (prefer) positive (negative) cokurtosis. In fact, the Cubic model estimation gives a positive estimate of α_2 and a negative estimate of α_3 for the Merger Arbitrage index. This means that both coskewness and cokurtosis diminish the possibilities for seeking risk compensation. As expected, the required rate of return for Merger Arbitrage in Table V is consistently smaller than the required return rate that the Market Model would entail. On the contrary, the Convertible Arbitrage, Emerging Market, Fund of Funds and Market Timing indices have positive cokurtosis and negative coskewness coefficients. Both risk components engender a larger required rate of return than the linear or two-moment market model.

Finally, comparing the historical and hypothetical scenarios, we note that required rates of return remain essentially unchanged for the two scenarios. Equity Non-Hedge is a special case. If the market portfolio return were 7.6% (historical scenario), the Equity Non-Hedge's required rate of return would be 14.49%. For a market portfolio return of 5.6% (hypothetical scenario), however, the required rate of return would decrease to 12.27%. Given that the Equity Non-Hedged index has the combination of the highest beta and lowest coskewness coefficients (see Table IV), this index has the widest exposure to market fluctuations. This explains an almost one-to-one relation between the required rate of return and the market portfolio return.

Conclusion

The main question investigated in this paper is how hedge fund index returns and market returns are related. We compare the traditional implementation of the market model relying on the mean-variance assumption with higher-moment market models that allow non-linear payoff patterns.

Our result suggests that each hedge fund, or hedge fund indices grouping together a common investment strategy, should be analysed separately since they are characterised by particular risk-return payoffs. The mere use of the two-moment market model for all hedge fund investment strategies may be misleading. In fact, a significant part of the hedge fund index performance is attributable to a non-linear relation between hedge fund index and market returns. This non-linear relation consists of systematic skewness and/or kurtosis. Hence, the multi-moment model is a flexible framework more suited to representing the variegate universe of the hedge fund industry.

One of the most instructive findings of this paper is that it highlights the relevance of coskewness and cokurtosis and not only of the beta. The lack of consideration of higher moments in pricing hedge funds in many cases leads to an insufficient compensation for the investment risk. For these hedge funds, negative coskewness makes a significant contribution to the investment return-risk profile and calls for a higher risk premium. On a simple historical data basis, we find that the highest additional premium due to coskewness – more than 4% on an annual basis – is for Distressed and Event Driven indices.

The empirical approach applied in this study relies purely on time series regression. Its most natural extension in future research is an empirical test of hedge funds' risk and returns in an equilibrium setting and hence a direct estimate of the traditional and higher-moment CAPM models.

Appendix 1

We estimate beta (systematic risk), gamma (coskewness), and delta (cokurtosis) using the method proposed by Kraus and Litzenberger [1976]. Once the regression of the Quadratic Model has provided estimates for α_1 and α_2, the procedure is as follows. First, take the expected values in equation (9) and subtract them to express the quadratic market model in deviation form; then multiply both sides by $\left(R_m - \overline{R}_m\right)$, taking expected values and dividing by σ_m^2. This yields an expression for beta, i.e.

$$\hat{\beta}_i = \hat{\alpha}_1 + \hat{\alpha}_2 \frac{S_m^3}{\sigma_m^2} \tag{A.1.1}$$

Similarly, multiplying both sides of the deviation form of the quadratic market model by $\left(R_m - \overline{R}_m\right)^2$, taking expected values and dividing by S_m^3, yields an expression for gamma, i.e.

$$\hat{\gamma}_i = \hat{\alpha}_1 + \hat{\alpha}_2 \frac{K_m^4 - \left(\sigma_m^2\right)^2}{S_m^3} \tag{A.1.2}$$

We extend this method to the Cubic Model. Consistent with the method above, we multiply both sides of the deviation form of the Cubic market model by $\left(R_m - \overline{R}_m\right)^3$, taking expected values and dividing by K_m^4 to yield an expression for delta. Thus, we get straightforward estimates for $\beta_{i,m}$, $\gamma_{i,m}$ and $\delta_{i,m}$ as shown above in equations (7).

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Table I: Descriptive statistics for monthly returns

This table shows the descriptive statistics of monthly returns for all hedge funds indices and their respective market portfolio. The sample period is from Jan-90 to Aug-02. The normality test is based on the Jarque-Berra statistics at 95%. The portfolio is composed of 70% Russell 3000 and 30% Lehman US aggregate. The Modified VaR at 99% accounts for volatility, skewness and kurtosis. It means that there is only 1% probability that the next monthly return will be lower than the given Modified VaR number.

	Mean	Volatility	Skewness	Excess kurtosis	Modified VaR 99%	Max loss	Normality
Convertible Arbitrage	0.9%	1.0%	-1.3	3.2	-2.4%	-3.2%	NO
Distressed Securities	1.2%	1.8%	-0.7	5.5	-6.1%	-8.5%	NO
Event Driven	1.1%	2.0%	-1.5	4.9	-6.4%	-8.9%	NO
Equity Hedge	1.5%	2.7%	0.1	1.1	-5.3%	-7.7%	NO
Emerging Markets	1.2%	4.6%	-0.8	3.4	-14.7%	-21.0%	NO
Equity Market Neutral	0.8%	1.0%	-0.1	0.2	-1.5%	-1.7%	NO
Equity Non-Hedge	1.3%	4.3%	-0.5	0.5	-10.3%	-13.3%	NO
Fixed Income Arbitrage	0.7%	1.4%	-1.7	8.7	-5.4%	-6.5%	NO
Fund of Funds	0.9%	1.7%	-0.3	3.7	-5.0%	-7.5%	NO
Macro	1.4%	2.5%	0.4	0.0	-3.4%	-3.8%	NO
Merger Arbitrage	0.9%	1.3%	-2.8	11.7	-4.5%	-6.5%	NO
Market Timing	1.1%	2.0%	0.1	-0.6	-3.1%	-3.3%	NO
Relative Value Arbitrage	1.1%	1.1%	-1.1	10.5	-4.6%	-5.8%	NO
Statistical Arbitrage	0.8%	1.2%	0.0	0.3	-1.9%	-2.1%	NO
Short Seller	0.6%	6.6%	0.1	1.0	-16.2%	-21.2%	NO
Weighted Composite	1.2%	2.1%	-0.7	2.6	-5.7%	-8.7%	NO
Portfolio	0.63%	3.1%	-0.6	0.8	-8.2%	-11.3%	NO
Wilshire All Growth	0.8%	4.1%	-0.9	2.4	-8.5%	-18.1%	NO
Merrill Lynch High Yield US	0.8%	3.7%	0.2	2.0	-6.1%	-12.9%	NO
US 1 Month Certif. Depos.	0.38%	0.1%	0.1	0.1	0.2%	0.1%	NO

Table II: the Market Model estimates for the Two-Moment CAPM

This table shows the regression coefficients from the Market Model estimation. In this model, Alpha0 is the constant and Alpha1 represents the systematic risk. The t-stat shows the statistical significance of the coefficients. A t-stat higher than 1.96 shows a significance level of 95%. For all hedge fund indices, the market portfolio is composed of 70% Russell 3000, 30% Lehman US bond aggregate except for Relative Value (Wilshire all growth) and Fixed income arbitrage (Merrill Lynch high yield US corporate). The risk free rate is the 1-month US Certificate of Deposit. The column on the right shows the adjusted R-square statistics.

		Alpha0	Alpha1	Adj.R2 Market Model
Convertible Arbitrage		0.0050	0.1255	
	t-stat	4.53	4.63	0.154
Distressed Securities		0.0069	0.2627	
	t-stat	3.40	5.36	0.194
Event Driven		0.0062	0.4453	
	t-stat	3.75	8.21	0.469
Equity Hedge		0.0090	0.6294	
	t-stat	5.54	13.25	0.510
Emerging Markets		0.0058	0.9009	
	t-stat	1.19	10.57	0.368
Equity Market Neutral		0.0045	0.0560	
	t-stat	6.01	1.67	0.035
Equity Non-Hedge		0.0068	1.1384	
	t-stat	3.24	22.10	0.691
Fixed Income Arbitrage		0.0028	0.0884	
	t-stat	1.78	2.33	0.057
Fund of Funds		0.0040	0.2683	
	t-stat	2.35	4.75	0.231
Macro		0.0093	0.3736	
	t-stat	3.93	7.27	0.218
Market Timing		0.0047	0.2125	
	t-stat	4.10	3.60	0.254
Merger Arbitrage		0.0062	0.4394	
	t-stat	4.73	8.41	0.468
Relative Value Arbitrage		0.0062	0.1163	
	t-stat	5.15	3.40	0.180
Statistical Arbitrage		0.0040	0.1981	
	t-stat	4.43	8.01	0.287
Short Seller		0.0058	-1.5922	
	t-stat	1.89	-12.72	0.556
Weighted Composite		0.0067	0.5226	
	t-stat	4.25	15.89	0.585

Table III: the Quadratic Model Estimates for the Three-Moment CAPM

This table shows the regression coefficients from the Quadratic Model estimation. In this model, Alpha0 is the constant, Alpha1 is the coefficient related to excess returns of the market portfolio, and Alpha2 refers to the market portfolio's unexpected returns squared. Using these alphas and the first four centred moments of the return distribution of the market portfolio, we calculate beta and gamma, which represent the systematic risk and coskewness implied by the Quadratic Model. Beta and gamma are reported in the last two columns on the right-hand side. The t-stat shows the statistical significance of the coefficients. A t-stat higher than 1.96 shows a significance level of 95%. For all hedge fund indices, the market portfolio is composed of 70% Russell 3000, 30% Lehman US bond aggregate except for Relative Value (Wilshire All Growth) and Fixed Income Arbitrage (Merrill Lynch High Yield US Corporate). the risk free rate is the 1-month US Certificate of Deposit. The two columns in the middle of the table show the adjusted R-square statistics for the Quadratic and Market models.

	Alpha0	Alpha1	Alpha2	Adj.R2 Quadratic Model	Adj.R2 Market Model	Implied β	Implied γ
Convertible arbitrage	0.0063	0.0984	-1.3371			0.125	0.282
t-stat	5.47	4.48	-2.22	0.198	0.154		
Distressed securities	0.0113	0.1703	-4.5726			0.260	0.797
t-stat	6.08	3.89	-4.92	0.339	0.194		
Event Driven	0.0104	0.3576	-4.3432			0.442	0.953
t-stat	7.36	9.46	-7.76	0.579	0.469		
Equity Hedge	0.0104	0.6000	-1.4521			0.628	0.799
t-stat	6.03	11.85	-1.96	0.517	0.510		
Emerging Markets	0.0114	0.7838	-5.7980			0.897	1.578
t-stat	2.51	7.91	-2.29	0.406	0.368		
Equity Market Neutral	0.0046	0.0545	-0.0744			0.056	0.065
t-stat	5.30	1.82	-0.11	0.035	0.035		
Equity Non-Hedge	0.0093	1.0860	-2.5936			1.137	1.441
t-stat	3.72	20.35	-3.05	0.700	0.691		
Fixed Income Arbitrage	0.0026	0.0864	0.2307			0.082	0.055
t-stat	1.74	2.22	0.54	0.059	0.057		
Fund of Funds	0.0060	0.2261	-2.0916			0.267	0.513
t-stat	3.87	3.61	-1.49	0.266	0.231		
Macro	0.0100	0.3597	-0.6909			0.373	0.454
t-stat	4.01	5.92	-0.70	0.220	0.218		
Market Timing	0.0073	0.1570	-2.7479			0.211	0.534
t-stat	7.08	3.85	-4.87	0.360	0.254		
Merger Arbitrage	0.0037	0.4920	2.6014			0.441	0.135
t-stat	2.62	13.43	3.82	0.509	0.468		
Relative Value Arb.	0.0085	0.0686	-1.3194			0.094	0.249
t-stat	7.89	3.50	-4.22	0.318	0.180		
Statistical Arbitrage	0.0035	0.2089	0.5350			0.198	0.136
t-stat	3.62	8.66	1.54	0.292	0.287		
Short Seller	0.0070	-1.6157	-1.1602			-1.593	-1.457
t-stat	1.78	-11.68	-0.54	0.557	0.556		
Weighted Composite	0.0093	0.4678	-2.7119			0.521	0.839
t-stat	6.00	16.70	-5.03	0.624	0.585		

Table IV: the Cubic Model estimates for the Four-Moment CAPM

This table shows the regression coefficients from the Cubic Model estimation. In this model, alpha0 is the constant, alpha1, alpha2 and alpha3 are the coefficients related to excess returns of the market portfolio and to the market portfolio's unexpected returns squared and cubed, respectively. Using these estimates and the first six centred moments of the return distribution of the market portfolio, we calculate beta, gamma, and delta, which represent the systematic risk, coskewness, and cokurtosis implied by the Cubic Model. Beta, gamma, and delta are reported in the last three columns on the right-hand side. The t-stat shows the statistical significance of the coefficients. A t-stat higher than 1.96 shows a significance level of 95%. For all hedge fund indices, the market portfolio is composed of 70% Russell 3000 and 30% Lehman US Bond Aggregate, except for Relative Value (Wilshire All Growth) and Fixed income arbitrage (Merrill Lynch high yield US corporate). The risk free rate is the 1-month US Certificate of Deposit. Three columns in the middle of the table show the adjusted R-square statistics for the Cubic, Quadratic and Market models.

	Alpha0	Alpha1	Alpha2	Alpha3	Adj.R2 Cubic M.	Adj.R2 Quadratic M.	Adj.R2 Market M.	Implied β	Implied γ	Implied δ
Convertible Arb.	0.0060	0.0653	-0.7222	12.4540				0.125	0.290	0.200
t-stat	5.23	1.96	-1.36	2.47	0.211	0.198	0.154			
Distressed Sec.	0.0112	0.1555	-4.2978	5.5654				0.260	0.800	0.430
t-stat	5.96	2.45	-5.14	0.55	0.340	0.339	0.194			
Event Driven	0.0104	0.3596	-4.3813	-0.7715				0.442	0.952	0.590
t-stat	7.36	7.24	-5.11	-0.10	0.579	0.579	0.469			
Equity Hedge	0.0106	0.6261	-1.9350	-9.7827				0.628	0.793	0.655
t-stat	5.92	10.26	-1.86	-0.93	0.518	0.517	0.510			
Emerging Markets	0.0099	0.5810	-2.0398	76.1224				0.898	1.626	1.275
t-stat	2.10	4.60	-0.83	2.74	0.429	0.406	0.368			
Equity Mkt Neutral	0.0044	0.0238	0.4944	11.5211				0.056	0.072	0.086
t-stat	4.83	0.64	0.82	1.58	0.047	0.035	0.035			
Equity Non-Hedge	0.0096	1.1380	-3.5575	-19.5235				1.136	1.429	1.180
t-stat	3.89	15.01	-2.91	-1.34	0.701	0.700	0.691			
Fixed Income Arb.	0.0024	0.1123	0.3036	-4.0632				0.092	0.030	0.065
t-stat	1.65	2.34	0.96	-1.75	0.065	0.059	0.057			
Fund of Funds	0.0056	0.1695	-1.0434	21.2305				0.267	0.526	0.389
t-stat	3.50	2.29	-0.82	1.50	0.279	0.266	0.231			
Macro	0.0100	0.3681	-0.8474	-3.1686				0.373	0.452	0.389
t-stat	3.97	5.22	-0.47	-0.16	0.220	0.220	0.218			
Market Timing	0.0070	0.1115	-1.9055	17.0621				0.211	0.544	0.345
t-stat	6.83	1.97	-2.70	2.40	0.374	0.360	0.254			
Merger Arbitrage	0.0045	0.5964	0.6661	-39.1984				0.441	0.111	0.260
t-stat	3.04	12.72	0.77	-4.09	0.541	0.509	0.468			
Relative Value Arb.	0.0081	0.0512	-0.8449	3.9249				0.082	0.206	0.127
t-stat	5.97	2.44	-1.51	1.30	0.327	0.318	0.180			
Statistical Arb.	0.0035	0.2064	0.5817	0.9474				0.198	0.136	0.182
t-stat	3.43	6.32	1.04	0.17	0.292	0.292	0.287			
Short Seller	0.0061	-1.7316	0.9884	43.5206				-1.592	-1.429	-1.451
t-stat	1.57	-8.67	0.34	1.06	0.561	0.557	0.556			
Weighted Comp.	0.0092	0.4657	-2.6728	0.7913				0.521	0.840	0.616
t-stat	5.70	10.84	-3.39	0.08	0.624	0.624	0.585			

Table V: Required Rates of Return for different hedge fund strategies

RRR stands for Required Rate of Return. The pricing models listed in the second column are the most appropriate specification according to the lowest Akaike Information Criterion (AIC) from the estimation of the Market, Quadratic, and Cubic Models. The Premium1 (Premium2) is the difference between the RRR1 (RRR2) and the return calculated on the basis of the Market Model. A positive premium is the higher required return the investor should require in order to bear significant negative coskewness and/or significant positive excess kurtosis. Two scenarios are analysed. The former (latter) implies an annual market portfolio return of 7.6% (5.6%) and a risk free rate of 4.6%, as from the historical data. The required rates of return and risk premia related to the former (latter) scenario are in column 6 and 7 (8 and 9) and they are dubbed RRR1 and Premium1 (RRR2 and Premium2).

	Model	AIC Market M.	AIC Quadratic M.	AIC Cubic M.	RRR1	Premium1	RRR2	Premium2
Convertible Arbitrage	Cubic	-6.530	-6.569	**-6.573**	7.43%	1.06%	7.30%	1.18%
Distressed	Quadratic	-5.329	**-5.514**	-5.502	14.03%	4.94%	13.68%	5.11%
Event-Driven	Quadratic	-5.573	**-5.793**	-5.780	13.50%	4.69%	12.79%	4.85%
Equity Hedge	Market	**-5.047**	-5.047	-5.036	12.65%	0.00%	11.42%	0.00%
Emerging Markets	Cubic	-3.750	-3.799	**-3.824**	13.60%	3.92%	12.46%	4.54%
Equity Market Neutral	Market	**-6.515**	-6.502	-6.502	5.58%	0.00%	5.47%	0.00%
Equity Non-Hedge	Quadratic	-4.625	**-4.641**	-4.634	14.34%	2.80%	12.21%	2.90%
Fixed Income Arbitrage	Market	**-5.783**	-5.772	-5.766	3.82%	0.00%	3.50%	0.00%
Fund of Funds	Cubic	-5.511	-5.544	**-5.548**	7.17%	1.61%	6.83%	1.80%
Macro	Market	**-4.774**	-4.763	-4.750	12.29%	0.00%	11.56%	0.00%
Market Timing	Cubic	-6.103	-6.243	**-6.252**	8.73%	2.44%	8.50%	2.64%
Merger Arbitrage	Cubic	-5.599	-5.665	**-5.720**	7.15%	-1.60%	5.98%	-1.91%
Relative Value Arbitr.	Quadratic	-6.329	**-6.501**	-6.500	10.60%	2.60%	10.30%	2.80%
Statistical Arbitrage	Market	**-6.409**	-6.403	-6.390	5.39%	0.00%	5.01%	0.00%
Short Seller	Market	**-3.377**	-3.365	-3.360	2.31%	0.00%	5.43%	0.00%
Weighted Composite	Quadratic	-5.723	**-5.808**	-5.795	12.48%	2.93%	11.56%	3.03%

Endnotes

[1] Source: www.hfr.com.

[2] The second derivative of the quadratic utility function increases with wealth. As the investor receives more money, he becomes more risk averse towards losses.

[3] Other interesting research has been done in this area. Amin and Kat [2002a] analyse portfolio volatility, skewness, and kurtosis with regard to a fund of hedge funds. Amin and Kat [2002b] estimate the cost of eliminating negative skewness. Ang, Chen and Xing [2001] investigate whether coskewness is significantly priced in stocks.

[4] Ineichen [2000] shows that, in 1999, the minimum investment in a hedge fund was between USD 100,000 and 1,000,000.

[5] See Elton and Gruber [1991].

[6] See Jurczenko, Maillet [2002].

[7] See Hwang and Satchell [1999, Appendix 2] for a complete derivation of the maximisation of the utility function with the Lagrangien.

8 This result is in line with Hwang and Satchell [1999].

9 Cf. also Jurczenko, Maillet [2002]

10 Van Royen [2002] uses 60% equity and 40% bonds for a local correlation analysis. Amin and Kat [2002a], [2002b] use a portfolio of 50% SP500 and 50% Salomon Brothers government bond index 10 years.

[11] The existence of higher moments in the hedge fund return distribution may be questioned. See the literature referring to the "tail index", e.g. Hsieh [1999]. Empirical evidence in Table I shows the relevance of moments higher than the second moment for the hedge fund return distributions.

[12] For the Modified VaR formula, see Favre and Galeano [2002]. The Modified VaR, an extension of the classical Value-at-Risk, accounts for return asymmetries (i.e. skewness) and fat tails (i.e. kurtosis). All else being equal, an asset with a negative skewness will have a lower Modified VaR than the classical Value-at-Risk.

[13] See Sharpe [1964], Lintner [1965] and Mossin [1966].

[14] See Samuelson [1970] for an exhaustive analysis.

[15] Further tests have been performed using the GMM method. As instrumental variables, we used the past returns. The GMM results basically support the LS results.

[16] This additional analysis is available on request.

[17] The merger arbitrage index is a good example of how the return distribution of an asset may differ from the return distribution of a portfolio including that asset. In fact, the return distribution of the Merger arbitrage index is close to a normal distribution (see Table I). In contrast, the joint return distribution of the market timing index and the market portfolio is characterised by positive skewness and negative kurtosis.