Hedging risk spillovers in international equity portfolios

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Abstract

By defining risk spillover as the transmission of return variances and covariances from one asset to another, we propose a flexible model to perform various hedging strategies in an international equity portfolio. According to the risk management strategy, the portfolio risk is seen as a specific combination of realized variances / covariances based on high frequency data. Of particular interest are the risk spillovers of equities within the same sector (sector spillover) and from currencies to international equities (currency spillover). The forecasting analysis shows that hedging only sector and currency spillovers rather than full hedging is viable both in economic and statistical terms.

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1 Introduction

Risk management and portfolio allocation deal with two main principles: minimizing portfolio risk and efficient hedging strategies. The literature on the former principle goes back to Markowitz's theory on portfolio diversification, Markowitz (1952). In a realistic setting with transaction costs and illiquid markets, however, investors deal with practical questions such as how much should be hedged and how to implement hedging. In this paper, we propose a flexible approach to model equity portfolio risk involving exchange rate risks. This model lends itself to adaptable definitions of risk and various structures of hedging. Specifically, we refer to the volatility transmission between exchange rate and equity returns. This approach allows us to specify the interactions between variances / covariances of exchange rate and equity returns and how these relations evolve across time.

Our methodology differs from the traditional definition of hedging which focuses on the determination of optimal portfolio weights. Between the two extremes of full or no hedging, a variety of combinations of partial hedging are conceivable. The main goal of this paper is to provide an effective method to implement partial hedging. Facing the trade-off between diversification needs and hedging costs, we assume that our agent decides to target a well-defined configuration of portfolio risk. In particular, we take the standpoint of a representative US investor holding a diversified portfolio of international equity shares of companies active in different sectors. While all equities are quoted in US dollars, our representative investor is concerned by two main sources of risk: currency and sector risk. The former relates to the unexpected outcomes of foreign companies. The latter comes from the common exposure to sector-wide factors. Our model enables the representative investor to measure and hedge any combination of portfolio risk. Three main aggregations of the variance-covariance matrix are considered: all elements, the diagonal elements, and blocks of elements according to the companies' sectors. The economic meaning of the first approach is the total portfolio risk, while the second case refers to a fully diversified portfolio in which only the systematic risk matters. The block-wide approach captures the entire spectrum of partial hedging. Of particular interest is spillover risk, i.e. the danger of adverse transmissions of return variances and covariances from one specific asset category to another. In other terms, we consider how sector-specific and currency risks interact within and across asset classes and across time. These aspects are particularly relevant for international equity shares such as American Depositary Receipt (ADRs).

The characteristics of our model facilitate the implementation of a number of economic thoughts such as the idea of conditional hedging in Glen and Jorion (1993)[1].

This approach is consistent with the variety of investment funds in the financial industry and it makes viable a broad spectrum of risk management strategies. For instance, the investor can believe ex ante that an asset class such as hedge fund investments should by its very nature be poorly correlated with other (traditional) securities such as stocks and bonds. Another example is represented by the typical disconnection between currencies and equities, or even among sectors that may be pro-cyclical (e.g. manufacturing and resources) or counter-cyclical (e.g. pharmaceutical and health sectors).

From the technical side, our approach relies on the Wishart autoregressive model (WAR) proposed by Gourieroux et al. (2009). The model is based on a dynamic extension of the Wishart distribution. This specification is compatible with financial theory, satisfies the constraints on volatility matrices, has a flexible form and, most importantly, maintains the coefficients' interpretability. The main innovation proposed in this paper is the introduction of specific model parametrizations that allows the control of the dynamics of the spillover between variances by imposing a particular structure on the coefficient matrices. Hence we name the model block WAR. The use of block struc-

[1] In their seminal paper, Glen and Jorion (1993) show that the inclusion of hedging instruments in international portfolios has two main benefits: the portfolio volatility decreases and the Sharpe ratio increases. On the other hand, de Roon et al. (2003) show that these results holds only in two circumstances: in case of static hedging, when investors are very risk-adverse; and in the case of dynamic hedging, when it is conditional on specific variables such as the interest rate spread.

tures in parameter matrices is similar to that adopted in Billio et al. (2006), Billio and Caporin (2008), and Engle and Kelly (2008) that introduce a block structure for the correlation matrix. Similar approaches have been considered in Asai et al. (2008) for multivariate stochastic volatility models while Caporin and Paruolo (2008) present a more general spatial solution to the curse of dimensionality problem in multivariate volatility models that includes as a special case a block structure on the coefficient matrices. More specifically, we perform a forecasting analysis to evaluate the economic implications coming from spillover hedging. The forecasting ability of several model specifications is assessed both in statistical and economic terms. On the economic side, the crucial mechanism is that the optimized portfolio weights depend on how spillovers are structurally modeled. Consistent with the recent literature on realized volatility, the empirical analysis is based on ultra-high frequency data of 12 stocks quoted on US equity markets and 3 exchange rates over a period of seven years that includes the recent financial crisis.

2 Modeling variance spillovers

We now introduce the model used to analyze and forecast the sequence of realized variance/covariance matrix of the 15 assets in our study. We then describe the set of alternative parametric restrictions to the spillovers between variances that help to reduce the complexity of the model estimation, and might be motivated by some economic criterion (e.g. assets classification) or data-driven. As we mention in the introduction, we presume that realized covariance sequences are available and so will not tackle the problem of the optimal estimation of realized variances and covariances. Details on these aspects could be found in Zhang et al. (2005), Bandi and Russel (2008),Hansen and Lunde (2006) ,Barndorff-Nielsen et al. (2008a) and Zhang (2010), among others.

2.1 The Wishart autoregressive process

Once we have computed the series of realized variance/covariance matrices from the intra-day returns, it is of fundamental importance for our purpose to adopt a model which is feasible to estimate even with a large set of assets, that guarantees the positive definitiveness of the forecasted covariance matrix, and whose coefficients keep their interpretability. This last aspect will be even more important if the quality of covariance matrix forecasts would be measured according to an economic criterion (such as the returns on various optimized portfolios).

A model that satisfies the previously itemized requirement is the realized Wishart autoregressive model (WAR) of Gourieroux et al. (2009). Refer to this paper for the details on the formal definition and properties of the WAR model. As shown in Bonato et al. (2008), this model is particularly suitable to test for variance spillover between assets, and in their paper the authors propose four specifications to restrict the interaction between past variances/covariances with their contemporaneous values.

Denote by Y_t the time t (realized) covariance for a group of n assets. If the sequence of stochastic positive definite Y_t matrices is said to follow a Wishart process, the expected value of Y_{t+1} conditioned on the information up to time t read:

$$E_t\left(Y_{t+1}\right) = M Y_t M' + K\Sigma. \tag{1}$$

and we say that Y_t is a Wishart autoregressive process of order 1, WAR(1), denoted $W[K, M, \Sigma]$. The transition density of WAR(1) depends on the following parameters: K, the scalar degree of freedom , strictly greater than $n-1$ (the number of assets minus one); M, the $n \times n$ matrix of autoregressive parameters; and Σ, the $n \times n$ symmetric and positive definite matrix of innovation covariances. An important property of the Wishart distribution is that the matrices Y_t are positive definite if and only if $K \geq n$ and for a non-centered Wishart specification, the distribution of Y_t possesses a density function only when $K > n-1$ (hence the condition above). Thus, for $K < n-1$ no density can be defined and for $K < n$ the process Y_t is given by a sequence of singular covariance matrices with degenerate Wishart distribution (Muirhead, 1982). See Gourieroux et al.

(2009) for additional details on the derivation of the WAR process.

In general, WAR processes with higher autoregressive order p may be considered and the Wishart process can be easily extended to include more autoregressive lags. This is accomplished by replacing the conditioning matrix MY_tM' with any symmetric positive semi-definite function of $Y_t, Y_{t-1}, \ldots, Y_{t-p+1}$. However, when the autoregressive order is larger than 1, the interpretation of the Wishart process as the sum of squares of autoregressive Gaussian processes is no longer valid even for integer K. For a WAR(p) process, the equivalent of (1) reads:

$$E_t(Y_{t+1}) = \sum_{j=1}^{p} MjY_{t+1-j}M_j' + K\Sigma. \tag{2}$$

In the following, unless differently stated, we will refer only to WAR(1) specifications.

2.2 Interpretation of the coefficients

Let us consider the simple setting in which we are given an even number of assets N which we can cluster into two groups of size $n/2$. Under the assumption that the autoregressive matrix M is block diagonal, with M_{11} and M_{22} being $n/2 \times n/2$ matrices, it is straightforward to show that

$$\begin{pmatrix} M_{11} & \\ & M_{22} \end{pmatrix} \begin{pmatrix} Y_{11,t} & Y_{12,t} \\ Y_{21,t} & Y_{22,t} \end{pmatrix} \begin{pmatrix} M_{11}' & \\ & M_{22}' \end{pmatrix} = \begin{pmatrix} M_{11}Y_{11,t}M_{11}' & M_{11}Y_{12,t}M_{22}' \\ M_{22}Y_{21,t}M_{11}' & M_{22}Y_{22,t}M_{22}' \end{pmatrix},$$

which gives the expression for $E_t[Y_{t+1}]$ with the quantity $K\Sigma$ to be added. We see that constraining M to be a block diagonal matrix has the effect of limiting co-volatility spillover effects *between* different groups. Only past variance-covariance values of group 1 influence future values of group 1. The same holds for group 2.

In order to analyse in greater details the spillover effect *within* groups, we consider the following (2×2) covariance matrix Y_t, autoregressive matrix M, and innovation variance Σ:

$$Y_t = \begin{pmatrix} Y_{11,t} & Y_{12,t} \\ Y_{12,t} & Y_{22,t} \end{pmatrix}, M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}$$

The full WAR(1) model specifies the best prediction of Y_t, $E_{t-1}[Y_t]$ as:

$$E_{t-1}[Y_t] = \begin{pmatrix} a_1 Y_{11,t-1} + b_1 Y_{12,t-1} + c_1 Y_{22,t-1} + d_1 & a_2 Y_{11,t-1} + b_2 Y_{12,t-1} + c_2 Y_{22,t-1} + d_2 \\ - & a_3 Y_{11,t-1} + b_3 Y_{12,t-1} + c_3 Y_{22,t-1} + d_3 \end{pmatrix} \tag{3}$$

where a_j, b_j, c_j and d_j, $j = 1, \ldots, 3$ are scalar parameters, and d_j corresponds to K times the entries of Σ. By construction, the prediction is a symmetric semi-definite positive matrix for any Y_{t-1} which belong to $\mathcal{S}^+$, the set of symmetric positive definite matrices. To express it in terms of M we have:

$$\begin{cases} a_1 = m_{11}^2, & b_1 = 2m_{11}m_{12}, & c_1 = m_{12}^2, \\ a_2 = m_{11}m_{21}, & b_2 = m_{11}m_{22} + m_{21}m_{12}, & c_2 = m_{12}m_{22}, \\ a_3 = m_{21}^2, & b_3 = 2m_{21}m_{22}, & c_3 = m_{22}^2, \end{cases}$$

The effect of the past variances and covariances on the present volatility can be seen immediately. First, note that the full WAR model allows for spillover between variances and covariances. For instance, setting $m_{12} = 0$ implies that the conditional variance of the first asset depends only on its past shocks and that the second asset variance does not influence the conditional covariance. Put differently, a diagonal specification of M corresponds to the absence of spillovers between variances and covariances.

Those restrictions on the dynamic model are clearly related to non-causality restrictions concerning volatilities and covolatilities. Linear (in the Granger sense) and non-linear causalities are investigated and compared, for a bivariate WAR process, in Jasiak and Lu (2007). Gourieroux and Sufana (2007) characterize nonlinear causality hypothesis for model based on the conditional Laplace transform (the WAR process being one) and provide interpretations of the linear and quadratic causality in this framework.

In the case in which M is diagonal, i.e. when $m_{12} = m_{21} = 0$, we have

$$\begin{aligned} E_t[Y_{11,t+1}] &= m_{11}^2 Y_{11,t} + K\sigma_{11}, \\ E_t[Y_{12,t+1}] &= m_{11} m_{22} Y_{12,t} + K\sigma_{12}, \\ E_t[Y_{22,t+1}] &= m_{22}^2 Y_{22,t} + K\sigma_{22}, \end{aligned}$$

and thus each entry of Y_t depends only on its past values.

This very simple example in two dimensions helps us to identify the coefficients in M that plays a role in the spillover effect between variances. Using the delta method we can, in fact, easily compute the standard errors for a_i, b_i and c_i and thus evaluate which parameters are significant and check the appropriateness of the assumption of limited spillover. We will now present four different parametrizations for the WAR process that impose no or limited spillover.

2.3 Specifications of the block Wishart autoregressive model

To derive the block WAR model, we impose a set of restrictions on the matrix M. These restrictions come from a criterion allowing assets to be grouped. Some examples are given by the economic sector of the stocks entering into an equity portfolio, the type of assets entering into a diversified equity-bond portfolio, or the geographical reference areas of a group of assets. The main intuition behind asset grouping is that the clustered variables may share common patterns or common features, and that their variance-covariance dynamic is similar. In fact, we can presume that assets belonging to the same economic sector may have a similar reaction to market shocks/news, and are similarly affected by market movements.

Clearly, groups may be defined on a data-driven basis, such as referring to the dynamic properties of the series mean and/or variances, or on mixed criteria. The comparison of alternative methods for clustering financial assets is outside the scope of this paper and will not be considered. Some examples are given in Bauwens and Rombouts (2007), and Aielli and Caporin (2011). In the following we will use *a priori* defined groups for stocks (sectors or cyclicality as grouping criteria) and keep unchanged the group composed with exchange rates.

In a very general case, we assume that our portfolio consists of n stocks and currencies and that we can classify them into N groups, according to some economic criterion, as discussed in the previous section (such as the economic sector or the existence of common patterns in realized variances and covariances). The N groups have dimension n_i with $\sum_i n_i = n$. In addition, the assets are ordered following a group rule, that is, assets from 1 to n_1 belong to group 1, assets from $n_1 + 1$ to $n_1 + n_2$ belongs to group 2, and so on. Given this asset classification, the autoregressive matrix M may be partitioned as follows:

$$M = \begin{pmatrix} M_{11} & \cdots & M_{1N} \\ \vdots & M_{ii} & \vdots \\ \vdots & \vdots & M_{N-1,N} \\ M_{N1} & \cdots & M_{NN} \end{pmatrix},$$

where M_{ij} is a matrix of dimension $n_i \times n_j$.

Imposing a particular structure on the matrices M_{ij} enables us to control for the spillover of volatility between and within groups. The basic assumption we are making is that spillover effects are not present between assets belonging to different groups. Therefore, M is a block diagonal matrix with zero off-diagonal elements: $M_{i,j} = \mathbf{0}, \forall i \neq j$. The structure considered for the diagonal blocks M_{ii} will be of two types:

(1) M_{ij} is full

(2) M_{ij} is diagonal.

Specification (1) implies that there exist spillover effects in terms of variances/covariances within groups belonging to the same sector. Specification (2) consider the case in which there is no spillover effect at all, even within groups. This means that the dynamics of future variances / covariances are solely a function of their past values. Both approaches can be further restricted by imposing a unique scalar values for all the entries in $M_{i,i}$.

Thus, within each group, a single parameter regulates the effect of past co-volatilities on their future values.

Throughout our empirical analysis of the economic implication of such restriction in terms of forecasting, we will generally assume M to be block diagonal while keeping the full M as benchmark. An exception is represented by the matrix $M_{N-1,N}$. Assume, for ease of exposition, we are interested in evaluating the volatility spillovers between assets belonging to groups $N-1$ and N. The most natural way to accomplish this would be to consider them as a unique sector and the lower left diagonal block of M of size $n_{N-1}+n_N$. This specification has the drawback that spillover effects are bi-directional, i.e. assets belonging to group $N-1$ influence group N and vice versa. In a situation in which assets in $N-1$ are equities and those in N are currencies, this may not be an appropriate choice as it implicitly assumes that a currency's volatilities are affected by these particular equities' volatilities. Simple (and tedious) algebra shows that imposing a diagonal structure for $M_{N-1,N}$ solves this issue. In particular, and ad hoc in our empirical analysis, if the two groups have the same dimensions, and the parameter matrices are diagonal, then the volatility of assets in N influences only the volatility of the assets in $N-1$ which hold the same position within the block. This spillover effect exists only between variances and is mono directional from group N to $N-1$.

3 Estimation

Following the exposition in Gourieroux et al. (2009), we obtain an analogous identification result for the block WAR and block HAR-WAR model.

The first-order conditional moments can be used to calibrate the parameters in M and Σ, up to the sign and scale factor, respectively. As the first-order method of moments is equivalent to non-linear least squares, the estimator is defined as:

$$\left(\hat{M},\hat{\Sigma}^*\right)=\text{Argmin}_{M,\Sigma^*}S^2\left(M,\Sigma^*\right)$$

where

$$S^2(M,\Sigma^*) = \sum_{t=2}^{T}\sum_{i<j}\left(Y_{ij,t} - \sum_{k=1}^{n}\sum_{l=1}^{n} Y_{kl,t-1}m_{ik}m_{lk} - \sigma^*_{ij}\right)^2$$
$$= \sum_{t=2}^{T} \|vech(Y_t) - vech(MY_{t-1}M' + \Sigma^*)\|^2$$

and $\Sigma^* = K\Sigma$.

To estimate the degrees of freedom we follow the strategy proposed in Bonato (2009), which has been shown to be less sensitive to the presence of extreme events in the co-volatility process. Consider a portfolio allocation $\alpha \in \mathbb{R}^n$. We know that the unconditional distribution of Y_t is a $W(K, 0, \Sigma(\infty))$, a centered Wishart distribution. We can therefore easily show that

$$\alpha' Y_t \alpha \sim \text{Ga}\left(\frac{K}{2}, 2\alpha'\Sigma(\infty)\alpha\right), \tag{4}$$

i.e. the distribution of the portfolio with allocation α is a gamma distribution with the degrees of freedom K as shape parameter, (see also Meucci, 2005). An unbiased estimator of K can be obtained simply via maximum likelihood by fitting a gamma distribution to the process $\alpha' Y_t \alpha$.

4 Data and model specification

The dataset we employ in our analysis consists of 15 assets, 12 stocks and 3 exchange rates. Stocks data are provided at a 1-minute frequency by *Tickdata.com*. Exchange rates data are also provided at 1-minute frequency by *Disktrading.com*. The period we consider was from January 2, 2003 to December 31, 2009. Table 1 reports the name of the assets used in the analysis along with their sector and cyclicality, the currency of their financial statements and betas.

Factoring out weekends, holidays and days with shorter trading hours (e.g. July 3 or December 24) and matching all the days across the sample, we are left with 1759 trading days. Intraday returns are computed by taking the log differences of the 1-minute

Table 1: Grouping of the stocks according to their market sector/geographical location. The βs (for the period under analysis) are also reported, computed using the S&P index as market index.

Name	Tick symbol	Sector	Cyclicality	Statements*	$\hat{\beta}$
Boeing Co.	BA	Industrial	Cyclical	USD	**1.33**
Caterpillar Inc.	CAT				**1.79**
Fedex	FDX				**1.11**
International Business Machine	IBM	Information Technology	Cyclical	USD	0.78
Hewlett Packard Co.	HPQ				**1.09**
Texas Instrument Inc.	TXN				**1.18**
Kraft Food Inc.	KFT	Consumer Services	Anti-cyclical	USD	0.66
Procter Gamble Co.	PG				0.57
Time Warner Inc.	TWX				**1.41**
Sanofi-Aventis SA	SNY	Healthcare	Anti-cyclical	EUR	0.86
Glaxo Smith Kline	GSK			GBP	0.62
Novartis AG	NVS			CHF	0.85
EUR/USD			FX		
GBP/USD					
USD/CHF					

* Statements denotes the currency in which the company releases its financial statements and dividends.

prices and multiplying by 10^4. Daily returns are obtained as the difference of the logarithm of the closing and opening price of the day and multiplying by 100. Note that we do not consider the effect of overnight returns as we assume the representative agent opens and closes his position within the same trading day. This simplifying assumption is motivated by the difficulty in considering the overnight effect jointly on currency and stock trading, as forex is virtually traded around the clock and stocks are not and the overlapping trading hours of our portfolio coincide with stocks' trading hours. Summary statistics for the 1-minute and open-to-close returns on the 15 series are reported in Table 2.

The goal of the paper is to check whether there is economic loss in term of forecasting power and asset allocation performances if a full spillover hedging for a portfolio with international exposure is relaxed and only partial or zero spillover effects are considered.

The first task of our analysis consists in computing a measure of covariance between the assets making up the portfolio. We accomplish this following the recent developments in literature and exploiting the availability of high frequency data.

The trading day we construct runs from 9:35 AM (first observation) to 16:00 (last observation). Sampling every 5 minutes, we obtained 77 intraday returns which we used to construct the series of realized covariance matrices.

In the next step we compute the series of realized covariance matrices using the classical estimator presented in Andersen et al. (2003) and Barndorff-Nielsen and Shephard (2004) and used, for example, in de Pooter et al. (2006):

$$Y_t = \sum_{i=1}^{I} r_{t-1+ih,h} r'_{t-1+ih,h} \tag{5}$$

We indicate with Y_t the realized covariance matrix at time t in order to to be coherent with our previous notation and because the use of Σ would probably create confusion as Σ denotes the covariance matrix of the Gaussian vector underlying the WAR(1) model. $r_{t-1+ih,h} \equiv p_{t-1+ih} - p_{t-1+(i-1)/h}$ denotes the $(n \times 1)$ vector of returns for the i-th intraday period on day t, for $i = 1, \dots, I$, and with $n = 4$ the number of assets. I is the number

Table 2: Descriptive statistics of the 1-minute and daily(open-to-close and close-to-close) returns over the period January 2, 2003 - December 31, 2009. The means are scaled by 10^4, the remaining the statistics by 10^2

Stock	Mean	Max	Min	SD	Skewness	Kurtosis
1-minute						
BA	0.011	3.436	-4.765	0.090	0.106	45.252
CAT	0.007	3.141	-4.821	0.102	-0.007	49.032
FDX	0.011	3.071	-3.258	0.088	0.258	45.202
IBM	0.023	2.893	-2.146	0.073	0.129	38.623
HPQ	0.040	6.724	-11.931	0.096	-2.556	420.662
TXN	0.007	5.251	-4.209	0.111	0.230	55.760
KFT	-0.000	4.800	-10.013	0.074	-3.490	610.883
PG	0.018	2.774	-5.452	0.063	-1.005	158.148
TWX	0.024	3.901	-3.129	0.103	0.345	34.706
SNY	-0.006	10.471	-8.398	0.085	-1.710	811.158
GSK	-0.012	5.852	-5.084	0.074	0.902	349.135
NVS	-0.000	5.280	-3.105	0.063	0.595	320.593
EUR	-0.039	0.693	-1.059	0.049	-0.097	12.486
GBP	0.025	1.122	-0.636	0.046	0.117	16.377
CHF	0.029	1.024	-1.155	0.045	0.036	21.113
Daily O-to-C						
BA	0.041	7.531	-8.323	1.607	-0.024	5.781
CAT	0.026	10.776	-8.562	1.873	-0.054	6.299
FDX	0.042	8.779	-9.297	1.747	0.143	6.895
IBM	0.091	5.836	-6.861	1.236	-0.172	6.456
HPQ	0.155	9.804	-11.170	1.715	-0.059	6.900
TXN	0.028	9.663	-8.599	1.970	0.053	4.828
KFT	-0.001	6.467	-6.517	1.284	0.050	6.589
PG	0.070	7.444	-6.817	1.038	-0.188	9.044
TWX	0.092	11.763	-11.935	1.780	0.289	9.813
SNY	-0.024	12.783	-8.692	1.511	-0.076	9.425
GSK	-0.046	6.552	-8.778	1.253	-0.282	8.494
NVS	-0.001	6.131	-5.836	1.089	0.154	6.909
EUR	-0.019	2.293	-2.792	0.397	-0.298	8.192
GBP	-0.022	1.880	-2.560	0.376	-0.159	7.801
CHF	-0.030	1.881	-2.745	0.429	-0.343	6.840

of intraday intervals, each of length $h \equiv 1/I$. In our case, with a frequency of five minutes, $I = 76$. One shortcoming of the covariance matrix estimator we adopted is that it is not efficient in the presence of market microstructure noise and asynchronous trading (see for example Sheppard, 2006, Lunde and Voev, 2007, Barndorff-Nielsen et al., 2008b, Mancino and Sanfelici, 2008, among others). We think this does not represent an issue here as, first, we use very liquid assets that are traded in the same markets (CBoT for the futures and OTC for the currencies). This reduces the distortion induced by stale prices, non-homogenous trading time, data points irregularly spaced, asynchronism, different institutional features using different trading platforms or exchange systems. Secondly, as shown in Barndorff-Nielsen et al. (2008b) using intraday data of 10 stocks from the Dow Jones index, the estimated realized covariance matrices based on 5-minute returns are not significantly biased[2] (compared to the matrices constructed using the outer products of the open to close returns) even though realized kernels remain the preferred estimators. As already mentioned before, we did not consider overnight returns. Including overnight returns would affect only the volatility of stocks because currencies are traded 24 hours and their equivalent to the overnight returns would be the over-weekend return. Therefore we contend that adding overnight returns to only some components of the portfolio would cause distortion in the realized volatility of the portfolio itself.

4.1 Asset classes and portfolio construction

As shown in the previous sections, the ability to model spillover effects within or between assets requires that these assets be combined to form well-defined classes.

At first, the 15 series that form our hypothetical portfolio can be split in 3 groups:

- 9 US stocks traded at NYSE - company financial statements in USD

[2] For a given estimator, say $Y_t = Cov_t^{5m}$, Barndorff-Nielsen et al. (2008b), consider the difference $d_t = Cov_t^{5m} - Cov_t^{OtoC}$ where Cov_t^{OtoC} is the outer product of the open to close returns, which when averaged over many days provide an estimator of the average covariance between asset returns. The sample bias is computed as $\bar{d}$ and the robust variance as $\bar{e}^2 = \gamma_0 + 2\sum_{h=1}^{q}\left(1 - \frac{h}{q+1}\right)\gamma_h$, where $\gamma_h = \frac{1}{T-h}\sum_{t=1}^{n-h}\eta_t\eta_{t-h}$. Here $\eta_t = d_t - \bar{d}$ and $q = \text{int}\{4(T/100)^{2/9}\}$. Under the null hypothesis of no difference between the two estimators at one percent level $\left|\sqrt{T}\bar{d}/\bar{e}\right| < 2.326$ for each entry of Cov_t^{5m}.

- 3 Europe-located stocks traded at NYSE as ADR - company financial statements in local currency (EUR,GBP and CHF)
- 3 exchange rates: EUR, GBP and CHF all against USD.

Considering the 9 US and 3 non-US stocks, these can be further grouped according to their sector:

- Industrials - BA, CAT, FDX
- Information Technology - IBM, HPQ, TXN
- Consumer Service - KFT, TWX, PG
- Healthcare - SNY, GKS, NVS

Our first approach will thus consider these 4 combinations of stocks and the currencies as the groups forming the portfolio.

Industrials and Information Technology are generally considered to be pro-cyclical whereas Consumer Service and Healtcare are anti-cyclical. The last column in Table 1 shows the plot of the βs of these assets for the period under analysis calculated with respect to the S&P 500 as market index. All but one assets labeled as pro-cyclical display a β greater than 1, indicating that these assets are more sensitive to market movements. The same holds for the anti-cyclical assets, with 5 out of 6 βs being smaller than 1, i.e. these stocks are less risky compared to the market index. Therefore we decided to include in our analysis a second approach to group the portfolio components: cyclical, no-cyclical and currencies.

We take the point of view of a US investor investing in these 12 US-traded stocks. Beside the market specific risk, our representative investor is also exposed to country specific risk as the performance of the ADRs stock are also influenced by the exchange rate between the local currency and the US dollar. To enable this investor to reduce the risk induced by this currencies exposure, we assume that he includes in his portfolio the three exchange rates (all against the USD) of the countries in which the non-US firms are located: UK (Glaxo Smith Kline), France (Sanovis Aventis SA) and Switzerland (Novartis AG). Being long in ADRs implies that perfect hedging will be obtained by being long USD against the local currency. To allow our investor to have short selling constraints, he must be able to hedge his positions with positive weights. As USD is not the leading currency against EUR and GBP, in the empirical analysis we will simply take the reciprocal USD/GBP and USD/EUR when building our portfolio.

The question we want to answer is whether or not considering volatility spillovers between stocks belonging to different groups or within the same group delivers acceptable out-of-sample forecasts of the realized covariance matrix of the portfolio under analysis. In particular, we are interested in the effect of what we dub *spillover hedging*, that is, the risk protection obtained by accounting for adverse volatility transmission from currencies to the respective ADRs. From the seminal paper of Glen and Jorion (1993) it is well known that introducing hedging instruments in an internationally exposed portfolio sensibly reduces the portfolio volatility and delivers higher Sharpe ratios. In terms of volatility spillovers, it is plausible that shocks in currencies will also affect the USD-traded ADRs and the optimal portfolio weights will be consequently adjusted. Therefore, it is of great interest to find the possible gains in portfolio variance forecasting also when these specific spillovers are considered.

Each different specification of the forecasting model we employ, the WAR, leads to a particular dynamics of the volatility transmission to future values. This is achieved imposing a structure to the autoregressive matrix M. Based upon these last and previous considerations, we propose the following structure.

(1) M full

(2) M block diagonal

(3) M block diagonal and spillover hedging

(4) M diagonal

(5) M diagonal and spillover hedging.

Specification (1) assumes that spillover effects are common to all the 15 assets compounding the portfolio. In (2) we cluster assets according to their industrial sector (or cyclical/non-cyclical). That is, 5 (or 3) groups with spillover effects allowed within the group only. Specification (3) is different from (2) in that mono-direction spillover effect is permitted between currencies to ADRs only. In particular, any of the 3 ADRs volatility will be influenced *only* by the past volatility of the corresponding exchange rate (provided that stocks and currency are properly ordered in the covariance matrix). This coincides with the previously introduced (Section 2) diagonal matrix $M_{N-1,N}$. No effect between covariances is considered. In (4) only volatility spillovers (and no covariance spillovers) are considered. (5) is an enriched version of (4) as, like in (3), it does not ignore a possible spillover hedging effect between currencies and ADRs.

5 Economic evaluation of hedging spillover risk

The main ingredient to evaluate the benefit or losses associated with a limited hedging of spillover risk in our theoretical portfolio is the forecasted variance/covariance matrix. Spillover effect determines the effect of past values of variances/covariances on future variances/covariances belonging to the portfolio (full spillover), to the same asset class (limited spillover) or on themselves only (no spillover).

In our empirical application forecasts are made one period ahead and for an evaluation period from $t+1$ until the end of the sample. t is set to be equal to 250 (approximatively one trading year) and the estimation sample is then rolled forward employing a moving window of 250 trading days. At each point in time t a model is used to produce a forecast of the covariance matrix $\hat{Y}_{t+1}$ which is used to obtain the optimal vector of weights $\mathbf{w}_{t+1}$. At time $t+1$ portfolio returns are computed using the expected optimal weights (function of $\hat{Y}_{t+1}$) and the realized returns on the assets. An alternative approach would also be to introduce a forecasting equation for the returns, but a model involving the conditional mean is beyond the goal of our research, which focuses purely on the role of the variance.

5.1 Direct evaluation

A natural way to directly evaluate the implication of a limited spillover hedging in a international and asset-specific exposures in a pure forecasting perspective is to evaluate and compare the out-of-sample forecasted variance/covariance matrices provided by the models under analysis.

The approach we consider for the comparison of alternative WAR specifications factors in two loss functions for multivariate volatility models. Following Patton and Sheppard (2009), and Clements et al. (2009), we define the following two loss functions

$$L^1_{t+1,i} = \frac{1}{k^2} vec\left(\hat{Y}_{t+1,i} - Y_{t+1}\right)' vec\left(\hat{Y}_{t+1,i} - Y_{t+1}\right), \tag{6}$$

$$L^2_{t+1,i} = \text{trace}\left(\left(\hat{Y}_{t+1,i}\right)^{-1} Y_{t+1}\right) - \log\left(\left|\left(\hat{Y}_{t+1,i}\right)^{-1} Y_{t+1}\right|\right). \tag{7}$$

which are derived from Patton and Sheppard (2009). The function $L^1_{t+1,i}$ is a multivariate mean squared error function, while L^2_{t+1} is included in the class of robust loss functions defined in Patton and Sheppard (2009) and similar to a quasi likelihood loss function.

These loss functions are included in the class of consistent functions defined in Laurent et al. (2009).

To verify the null that $E\left[L^w_{i,t+1}\right] = E\left[L^w_{j,t+1}\right]$, $w = 1,2$ we could use a Diebold-Mariano-type test (see Diebold and Mariano, 1995, and West, 1996), computing the test statistic

$$d^w_{ij,t+1} = L^w_{i,t} - L^w_{j,t} \tag{8}$$

$$L^w_{ij} = \frac{\bar{d}^w_{ij}}{\sqrt{Var\left(\bar{d}^w_{ij}\right)}}$$

where $\bar{d}^w_{ij} = \frac{1}{W}\sum_{l=1}^{W} d^w_{ij,t+1}$, and $Var\left(\bar{d}^w_{ij}\right)$ is obtained by a HAC estimator. However, when many alternative models must be considered, other methods are needed. Some popular approaches are the Reality Check of White (2000), the Superior Predictive Ability test of Hansen (2005), and the Model Confidence Set (MCS) ofR. et al. (2003), R. et al. (2011).

We consider here the last method, which allows the creation of a set of models whose forecasting performances are statistically equivalent. The MCS uses as inputs all pairwise loss differentials $d^w_{ij,t}$ for a given loss function w and for all $i,j = 1,2,..P, i \neq j$, where P is the total number of fitted models. The MCS then proceeds by performing a sequential elimination procedure testing on a set of models $\mathcal{M}_l$ the following null hypothesis $H_0 : E\left[\bar{d}^w_{ij,t+1}\right] = 0$, with $i > j$ and for all $i,j \in \mathcal{M}_l$. The initial set $\mathcal{M}_1$ contains all models, and if the null hypothesis is rejected, the worst performing model is excluded from the set. Then the procedure works iteratively until the null is not rejected. Each step thus performs two operations at a generic iteration l: verify the null hypothesis and stop if accepted; if the null is rejected, identify the worst performing model and remove it from the set. To verify the null hypothesis Hansen et al. (2005) proposed two distinct test statistics based on the quantity (8) with the difference that the variance is computed using a bootstrap procedure (see R. et al. (2011) for details). The two statistics are the following

$$T_R = \max_{i,j \in \mathcal{M}_l} \left| L^w_{ij} \right|, \tag{9}$$

$$T_{SQ} = \sum_{i,j \in \mathcal{M}, i>j} \left(L^w_{ij} \right)^2. \tag{10}$$

Given that the covariances across the forecasts produced by the models included in a specific set are not null, the test statistics have non-standard and complicated distribution. To determine the p values of the test statistic a bootstrap approach has been proposed by Hansen et al. (2005). For a specific confidence level α, the null hypothesis can thus be verified by determining the bootstrapped p values. If it is rejected, the worst performing model is identified as

$$i = \arg\max_{i \in \mathcal{M}} \sum_{j \in \mathcal{M}} \bar{d}^w_{ij} \left(Var \left(\sum_{j \in \mathcal{M}} \bar{d}^w_{ij} \right) \right)^{-1}.$$

The MCS method was originally proposed for the comparison of univariate volatility forecasts but Patton and Sheppard (2009) suggest it could also be of interest in the multivariate framework, a claim supported by the analysis in Clements et al. (2009). In this paper the MCS will be used as a tool for comparing nested models with respect to their forecasting performances to verify the null hypothesis that a restricted model provides forecasts statistically equivalent to those produced by an unrestricted model.

5.2 Economic evaluation

A direct approach, although appealing and easy to implement, lacks of an economic interpretation of the result as the benefit of choosing the best forecasting model is not clear for the representative agent investing in the theoretical portfolio.

A more interpretable approach to discriminate between the plethora of models we can use and yet which provides a direct economic evaluation of the forecasting performance of the variance/covariance matrices is an indirect one. An indirect approach does not evaluate directly the ability of the covariance models to compute accurate forecasts, but consider economic criteria associated with the forecasting power of each model. In this paper we consider an asset allocation framework and compare the impact of model choice by contrasting the performance of specific portfolios:

- Equally weighted portfolio, denoted as EW, which is not exposed to the asset return mean estimation error and is superior to many other portfolios (see Miguel et al., 2007),

- Global minimum variance portfolio with and without short selling constraints, denoted as GMV and GMVB, respectively.

The weights of the equally weighted portfolios are $\mathbf{w} = \mathbf{1}_n/n$, where $\mathbf{1}_1$ denotes the n-dimensional vector of unit elements. The GMV weights are time and model-dependent. Their derivation is based on the covariance forecast:

$$\mathbf{w}_{t+h} = \frac{\hat{Y}_{t+1}^{-1}\mathbf{1}}{\mathbf{1}'\hat{Y}_{t+1}\mathbf{1}}, \tag{11}$$

with $\hat{Y}_{t+1}$ denoting the 1 step ahead forecasted covariance matrix based on the information set up to time t.

GMVB weights are determined by solving the optimization problem

$$\begin{aligned} \arg\min \quad & \mathbf{w}'\hat{Y}_{t+1}\mathbf{w} \\ \text{s.t.} \quad & w_j \geq 0 \quad j = 1,\ldots,n \\ \text{and} \quad & \mathbf{w}'\mathbf{1} = 1 \quad . \end{aligned} \tag{12}$$

For the different variance forecasting models and portfolio allocation weights we then define the following quantities:

(A1) Realized portfolio returns : $r_{t+1} = \boldsymbol{\omega}_{t+1}\mathbf{r}_{t+1}$,

(A2) Expected portfolio returns : $\hat{r}_{t+1} = \mathbf{w}_{t+1}\mathbf{r}_{t+1}$,

(B1) Realized portfolio variances : $\sigma^2_{t+1} = \boldsymbol{\omega}' Y_{t+1} \omega$,

(B2) Expected portfolio variances : $\hat{\sigma}^2_{t+1} = \mathbf{w}' \hat{Y}_{t+1} \mathbf{w}$,

(C1) Realized Sharpe ratio : $S_{t+1} = \frac{r_{t+1}}{\sqrt{(s_{t+1})}}$,

(C2) Expected Sharpe ratio : $\hat{S}_{t+1} = \frac{\hat{r}_{t+1}}{\sqrt{(h_{t+1})}}$,

where $\boldsymbol{\omega}$ denotes the ex-post solution to the optimization problem in equation (12), i.e. when the realization of Y_{t+1} is observed. A1, B1 and C1 represent a sort of 'Oracle prediction' as used in Chiriac and Voev (2010).

The methods we apply for the indirect model evaluation use the MSE and QLIKE univariate loss function

$$lf^1_{t+1} = (\hat{\sigma}^2_{t+1} - \sigma^2_{t+1})^2, \tag{13}$$

$$lf^2_{t+1} = \log(\hat{\sigma}^2_{t+1}) + \sigma^2_{t+h}\hat{\sigma}^{-2}_{t+1} \tag{14}$$

$$lf^3_{t+1} = (\hat{S}_{t+1} - S_{t+1})^2, \tag{15}$$

Loss functions (13) and (14) are similar to those adopted in the direct model evaluation and consider as target the ex-post realized portfolio volatility, i.e. the one computed after the optimal covariance matrix is known and optimal weights are computed. Loss function (15) is slightly different in the sense that it considers what we dubbed *realized Sharpe ratio*. Since in every point in time we observe both returns and volatility, it makes sense to construct a series of Sharpe ratios which do not arise simply as the ratio between total expected returns and sample portfolio volatility, but which provides for every forecasted value the ratio between expected (i.e. forecasted) portfolio returns and expected portfolio volatilities. Both expected returns and expected portfolio volatility are strictly dependent on the forecasted covariance matrix. Therefore our target will not be to maximize this expected Sharpe ratio but minimize the distance to this ex-post realized ratio. The indirect evaluation of the models then proceeds using the MCS approach previously discussed.

5.3 Empirical results

The results of the out-of-sample forecasting exercise are reported in Table 3 and 4 for the direct and economic evaluation respectively. Each entry reports for the 5 models considered, the average values of the loss function used. Cells marked grey indicate that the corresponding model belong to the 5% MCS. Along with the values of the loss function computed on the entire sample, we also reported the results for subperiods having year 2003 as starting estimation window and then covering years from 2004 to 2008. This was done to check if the recent financial crisis which peaked in early 2009 had any influence on the choice of the optimal model.

Although the evidence is not striking, the direct evaluation of the covariance matrix forecasts display a preference for the diagonal WAR with currency-ADR spillover effects. Loss function L^1 tends to accept all the models as the best models, with some exception for the full WAR. On the other hand, the quasi-likehood loss function L^2 only accepts the diagonal WAR with spillover within the set of best performing models. Note that these results are robust to the different periods under analysis.

When an equally-weighted portfolio is constructed, there is no clear outperforming model when the volatility MSE loss function is used, nor for the realized Sharpe ration MSE. With the QLIKE univariate loss function it is preferable to choose the most parameterized models: the full, the block diagonal and the block diagonal with spillovers. The inclusion of the block diagonal models highlights the importance of considering spillovers within groups. This might be due to the fact that, being the weights fixed a priori, more information is conveyed in the forecasts by more structured models. This is the opposite of the results obtained when using GMV and the GMV with portfolios optimized with short selling constraints. These two approaches are in fact coherent in preferring the least parameterized models (diagonal), with or without spillover effects between currencies and ADRs. Results are robust to the choice of the MCS statistics and through the different samples. Note, however, that the diagonal model which take into account the volatility transmission from currencies to ADRs is generally preferred to its simpler counterpart. This might be explained in the following way: It is well known that parameterized models in an optimal portfolio choice an carry estimation error within the optimization procedure. The impact of the estimation error can be so strong that recent studies (e.g. Miguel et al., 2007) argue there is no model that can beat the simple equally weighted portfolio. Therefore, in our case, a trade-off arises between the number of parameters in the model to minimize the estimation error and the ability to effectively capture the dynamics of the co-volatilities and the volatility spillover transmissions. The diagonal model with spillover effect is proven to be the best to solve this trade-off. On the one hand it is parsimonious, as only volatility spillovers between stocks are not considered (M diagonal). On the other hand, it accounts for the volatility transmission between currencies and correspondent ADR, capturing features that the simple diagonal model des not.

A different interpretation to the results obtained in the analysis, with particular focus on the GMV and constrained GMV allocation is to be related to the very nature of our theoretical portfolio. Although grouped according to their sector or their cyclicality, assets under consideration represent a good example of a well diversified portfolio. Therefore it comes as no surprise that spillover effects are evident only between ADRs and FX and mixed within the rest of the sectors. ADRs, the price of which is directly influenced by the exchange rate between the US dollar and the domestic currencies, are affected by shocks on exchange rate volatilities. This gains more importance in a global minimum variance optimization framework and our results corroborate this argumentation. Had groups been used, built with, for example, banking, internet service or air transportation stocks which are by their nature correlated to each other, the importance of considering volatility spillover would have certainly emerged. In this paper we focused our attention on the risks involved in an internationally exposed portfolio. We therefore employ a realistic, rather well-diversified, theoretical portfolio whose components can still be combined according to an economically meaningful definition. The true object of interest is then judging the benefits of including spillover effects between currencies and equity shares of foreign companies.

5.4 Robustness check

To corroborate our results and to assess the role played by the latest financial crisis, we performed a robustness check by splitting the data into 3 subgroups. Keeping the starting point fixed (year 2003), we only considered forecasted values up to 2006, 2007 and 2008 respectively. These subgroups consider the years previous to and during the crisis. As done before, we employed direct and indirect criteria to evaluate the impact of assuming limitation in the volatility transmission dynamics. Results are reported in Tables 5 to 8. Figures point as the same direction as obtained considering the whole sample. Again, pure statistical methods tend to prefer a diagonal model which also considers spillover effects between currencies and ADRs. When economic criteria are considered, a passive portfolio (with no optimized weights) does not particularly discriminate between models (loss function L^1) or tend to select the less parsimonious models (full or block diagonal). Optimized portfolios (with or without short selling constraints) on the contrary favor the least parametrized models (diagonal) but also highlight the importance of considering the volatility transmission between exchange rates and ADRs.

6 Conclusions

In this paper we investigated the benefits of hedging volatility spillover risks in a internationally exposed, well-diversified, portfolio. Spillover risk is defined as the danger of adverse transmissions of return variances and covariances from one specific asset category to another. We focused in particular on the volatility transmission between foreign stocks traded at NYSE (as ADR) and their home country's exchange rates (against the US dollar). The latter are used as hedging instruments against currency volatility risks.

We dubbed this aspect of risk protection *spillover hedging*.

We first introduced a set of specific parameterizations that allow control of the dynamics of the spillover between variances by imposing a particular structure on the model's coefficients. Our approach relies on the Wishart autoregressive model (WAR) proposed by Gourieroux et al. (2009). The model is based on a dynamic extension of the Wishart distribution. This specification is compatible with financial theory, satisfies the constraints on volatility matrices, has a flexible form and, most importantly, maintains the coefficients' interpretability

The assessment of the economic implications of spillover hedging is then performed in a forecasting framework. The forecasting ability of several model specifications is evaluated both in statistical and economic terms. On the economic side, the crucial mechanism is that the optimized portfolio weights depend on how spillovers are structurally modeled. Consistent with the recent literature on realized volatility, the empirical analysis is based on ultra-high frequency data of 12 stocks quoted on US equity markets (3 are ADRs) and 3 exchange rates over a period of seven years that includes the recent financial crisis. As a result, the forecasting analysis shows that hedging only sector and currency spillovers rather than full hedging is viable both in economic and statistical terms.

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Table 3: Average values of the direct loss functions implemented for the different portfolios. A grey cell indicates that the model belongs to the 5% MCS according to both T_R and T_{SQ}.

Model	L^1	L^2	L^1	L^2
Clustering	Sectorial		Cyclical	
Full	4.155	277.943	4.155	277.943
Diagonal	3.188	30.983	3.187	30.983
Block diagonal	3.448	41.142	3.536	53.601
Diagonal spill.	3.177	21.911	3.176	21.910
Block diag. spill.	3.431	46.512	3.603	22.362

Table 4: Average values of the direct loss functions implemented for the different portfolios. A grey cell indicates that the model belongs to the 5% MCS according to both T_R and T_{SQ}. ^{a} and ^{b} indicates that them model belongs to the 5% MCS according only to T_R or T_{SQ}, respectively.

Model	1/N			GMV			GMVr		
Sectorial	lf^1	lf^2	lf^3	lf^1	lf^2	lf^3	lf^1	lf^2	lf^3
Full	1.878	0.028	0.074	0.029	-2.696	12.653	0.095	-2.417	5.140
Diagonal	1.347	0.044	0.084	0.000	-3.593	2.114	0.000	-3.448	1.515
Block diagonal	1.495	0.026	0.079	0.000	-3.377	3.046	0.000	-3.261	2.040
Diagonal spill.	1.351	0.042	0.083	0.000	-3.686	1.789	0.000	-3.538	1.275
Block diag. spill.	1.493	0.025	0.078	0.000	-3.525	4.109	0.000	-3.375	1.991
Cyclical									
Full	1.878	0.028	0.074	0.029	-2.696	12.653	0.095	-2.417	5.140
Diagonal	1.347	0.044	0.084	0.000	-3.593	2.114	0.000	-3.448	1.515
Block Diagonal	1.441	0.021	0.077	0.003	-2.954	5.483	0.005	-2.858	3.936
Diagonal spill.	1.351	0.042	0.083	0.000	-3.686	1.789	0.000	-3.538	1.275
Block diag. spill.	1.594	0.013	0.073	0.000	-3.668	1.866	0.000	-3.532	1.316

Table 5: Average values of the direct loss functions implemented for the different portfolios and the sectorial cluster. A grey cell indicates that the model belongs to the 5% MCS according to both T_R and T_{SQ}.

Model	L^1	L^2
2003-2006		
Full	0.427	56.109
Diagonal	0.382	31.129
Block diagonal	0.384	40.770
Diagonal spill.	0.380	21.885
Block diag. spill.	0.385	37.559
2003-2007		
Full	0.409	54.349
Diagonal	0.369	30.458
Block diagonal	0.372	39.236
Diagonal spill.	0.366	21.941
Block diag. spill.	0.371	43.737
2003-2008		
Full	4.469	210.381
Diagonal	3.522	31.174
Block diagonal	3.825	41.292
Diagonal spill.	3.517	22.043
Block diag. spill.	3.812	50.931

Table 6: Average values of the direct loss functions implemented for the different portfolios and the sectorial cluster. A grey cell indicates that the model belongs to the 5% MCS according to both T_R and T_{SQ}. [a] and [b] indicates that them model belongs to the 5% MCS according only to T_R or T_{SQ}, respectively.

Model	1/N			GMV			GMVr		
Sectorial	lf^1	lf^2	lf^3	lf^1	lf^2	lf^3	lf^1	lf^2	lf^3
2003-2006									
Full	0.020	-0.472	0.056	0.001	-3.174	5.624	0.001	-3.105	4.358
Diagonal	0.018	-0.476	0.055	0.000	-3.884	1.403	0.000	-3.805	1.129
Block diagonal	0.016	-0.485	0.053	0.000	-3.521	3.351	0.000	-3.471	2.373
Diagonal spill.	0.018	-0.475	0.055	0.000	-3.925	1.350	0.000	-3.859	0.913
Block diag. spill.	0.016	-0.485	0.054	0.000	-3.794	2.321	0.000	-3.724	1.688
2003-2007									
Full	0.031	-0.436	0.068	0.001	-3.183	4.959	0.001	-3.105	3.694
Diagonal	0.031	-0.430	0.073	0.000	-3.895[a]	1.400	0.000	-3.782	1.114
Block diagonal	0.030	-0.442	0.073	0.000	-3.544	2.883	0.000	-3.479	1.988
Diagonal spill.	0.031	-0.431	0.073	0.000	-3.950	1.172	0.000	-3.844	0.823
Block diag. spill.	0.029	-0.443	0.072	0.000	-3.802	3.030	0.000	-3.705	1.437
2003-2008									
Full	2.064	-0.137	0.074	0.016	-2.905	7.757	0.053	-2.737	3.893
Diagonal	1.518	-0.126	0.084[a]	0.000	-3.719	1.284	0.000	-3.576	1.020
Block diagonal	1.706	-0.141	0.080	0.000	-3.412	2.676	0.000	-3.314	1.826
Diagonal spill.	1.522	-0.128	0.083	0.000	-3.758	1.285	0.000	-3.621	0.903
Block diag. spill.	1.702[b]	-0.143	0.079	0.000	-3.635	3.726	0.000	-3.507	1.463

Table 7: Average values of the direct loss functions implemented for the different portfolios and the cyclical cluster. A grey cell indicates that the model belongs to the 5% MCS according to both T_R and T_{SQ}.

Model	L^1	L^2
2003-2006		
Full	0.427	56.109
Diagonal	0.382	31.129
Block diagonal	0.395	34.424
Diagonal spill.	0.380	21.885
Block diag.spill.	0.390	22.035
2003-2007		
Full	0.409	54.349
Diagonal	0.369	30.458
Block diagonal	0.383	33.777
Diagonal spill.	0.366	21.941
Block diag.spill	0.378	22.106
2003-2008		
Full	4.469	210.381
Diagonal	3.522	31.174
Block diagonal	3.869	47.885
Diagonal spill.	3.517	22.043
Block diag.spill.	3.986	22.369

Table 8: Average values of the direct loss functions implemented for the different portfolios and the cyclical cluster. A gray cell indicates that the model belongs to the 5% MCS according to both T_R and T_{SQ}. ^{a} and ^{b} indicates that them model belongs to the 5% MCS according only to T_R or T_{SQ}, respectively.

Model	1/N			GMV			GMVr		
Sectorial	lf^1	lf^2	lf^3	lf^1	lf^2	lf^3	lf^1	lf^2	lf^3
2003-2006									
Full	0.020	-0.472	t0.056[a]	0.001	-3.174	5.624	0.001	-3.105	4.358
Diagonal	0.018[a]	-0.476	0.055[a]	0.000	-3.884[a]	1.403	0.000	-3.805[a]	1.129
Block diagonal	0.017	-0.484	0.054[a]	0.001	-3.215	5.506	0.001	-3.168	4.316
Diagonal spill.	0.018[a]	-0.475	0.055[a]	0.000	-3.925	1.350	0.000	-3.859	0.913
Block diag.spill.	0.016	-0.490	0.050	0.000	-3.924	1.358	0.000	-3.858	0.922
2003-2007									
Full	0.031	-0.436	0.068	0.001	-3.183	4.959	0.001	-3.105	3.694
Diagonal	0.031	-0.430	0.073	0.000	-3.895	1.400	0.000	-3.782	1.114
Block diagonal	0.030	-0.443	0.073	0.001	-3.241	4.763	0.001	-3.189	3.626
Diagonal spill.	0.031	-0.431	0.073	0.000	-3.950	1.172	0.000	-3.844	0.823
Block diag.spill.	0.029	-0.450	0.070	0.000	-3.952	1.174	0.000	-3.853	0.803
2003-2008									
Full	2.064	-0.137	0.074	0.016	-2.905	7.757	0.053[a]	-2.737	3.893
Diagonal	1.518	-0.126	0.084[a]	0.000	-3.719	1.284	0.000	-3.576	1.020
Block diagonal	1.637	-0.146	0.078	0.002	-3.079	4.662	0.003[a]	-3.006	3.318
Diagonal spill.	1.522	-0.128	0.083	0.000	-3.758	1.285	0.000	-3.621	0.903
Block diag.spill.	1.825	-0.153	0.074	0.000	-3.761	1.267	0.000	-3.634	0.859